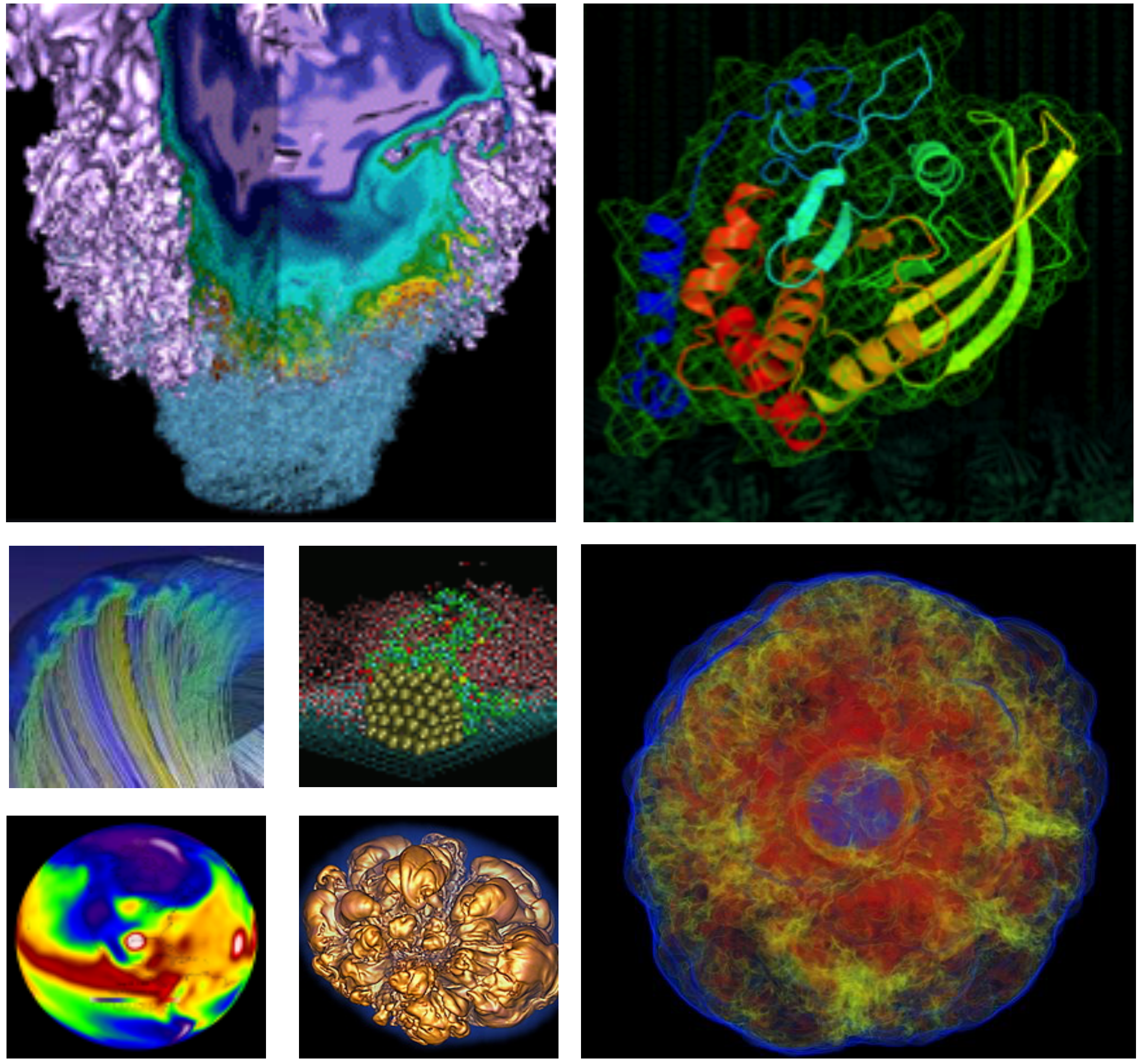
S9624:

Performance Analysis of GPU-Accelerated Applications using the Roofline Model

GTC 2019, San Jose



Charlene Yang
Application Performance Specialist
NERSC, LBNL
cjyang@lbl.gov

Samuel Williams
Senior Staff Scientist
CRD, LBNL
swwilliams@lbl.gov

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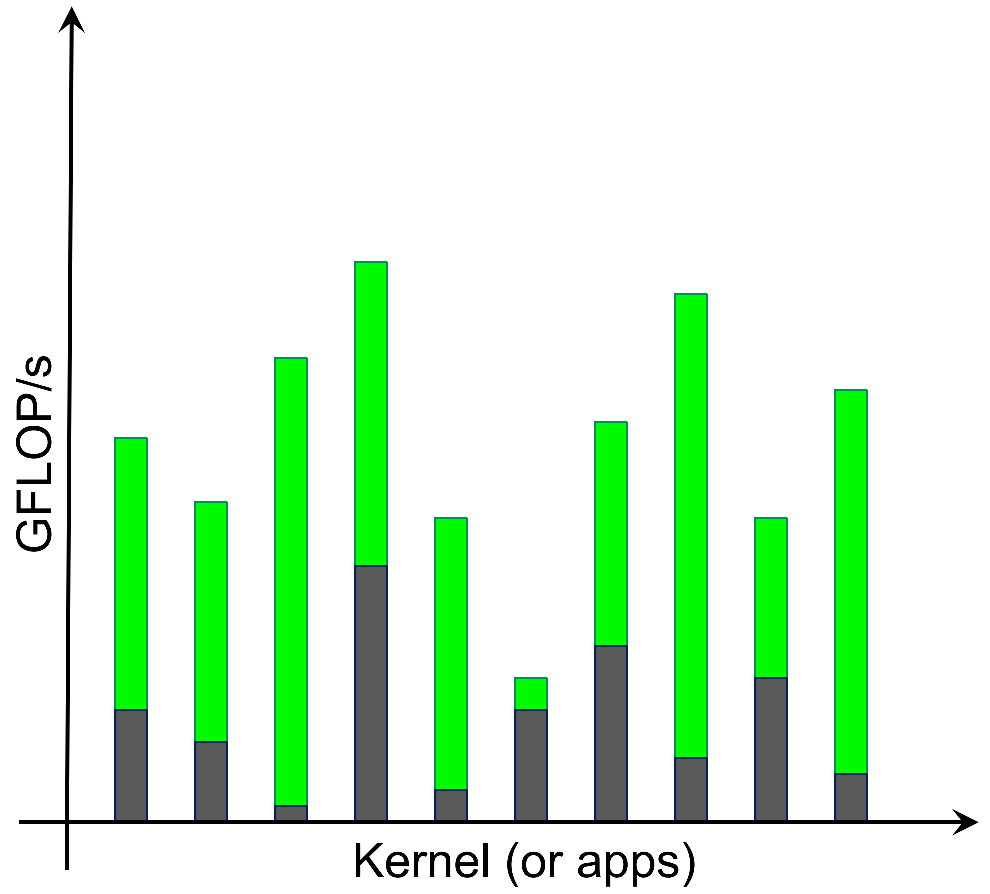
You just bought a $10,000 throughput-optimized GPU!

Are you making good use of your investment?

You could just run benchmarks

- Imagine a mix of benchmarks or kernels…
- GFLOP/s alone may not be particularly insightful
- Moreover, speedup relative to a Xeon may seem random



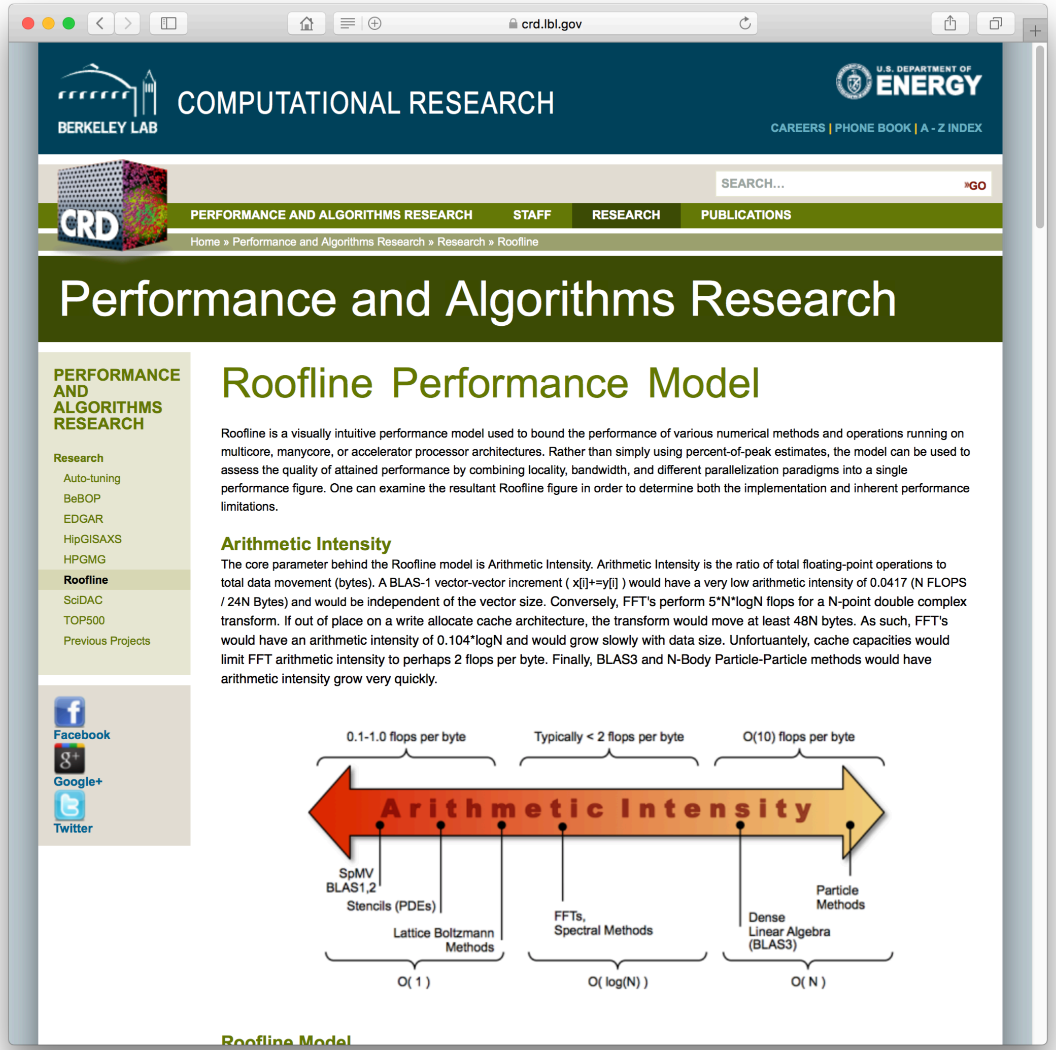
Making good use of your GPU?

1. Are you operating it in the throughput-limited regime?
   - Not sensitive to Amdahl effects
   - Not sensitive to D2H/H2D transfers
   - Not sensitive to launch overheads
   - Not sensitive to latencies

2. If in the throughput-limited regime, are you making good use of the GPU's **compute** and **bandwidth** capabilities?

The Roofline Model

- **Roofline Model** is a throughput-oriented performance model
- Premised on the interplay between FLOP/s, bandwidth, and reuse
- Tracks <u>rates</u> not times
- Independent of ISA and architecture (applies to CPUs, GPUs, Google TPUs, etc…)

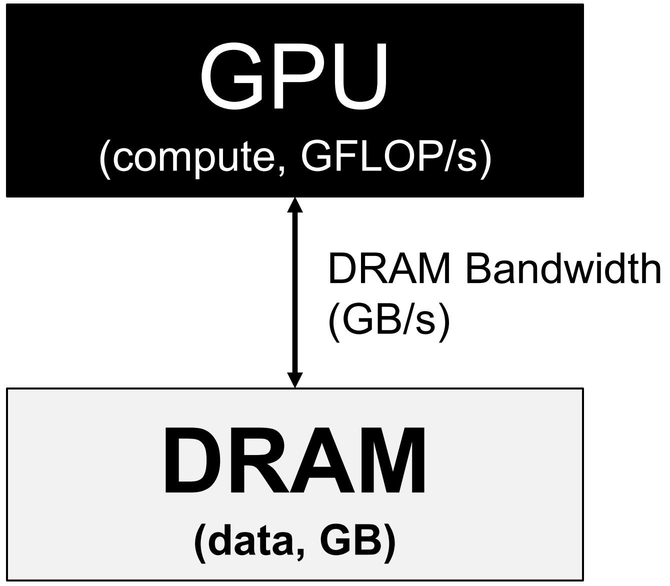


https://crd.lbl.gov/departments/computer-science/PAR/research/roofline

Jouppi et al, "In-Datacenter Performance Analysis of a Tensor Processing Unit", ISCA, 2017.

(DRAM) Roofline

- One could hope to always attain peak performance (GFLOP/s)
- However, finite locality (reuse) and bandwidth limit performance.
- Assume:
  - Idealized processor/caches
  - Cold start (data in DRAM)



$$\textbf{Time} = \max \begin{cases} \text{\#FLOPs / Peak GFLOP/s} \\ \text{\#Bytes / Peak GB/s} \end{cases}$$

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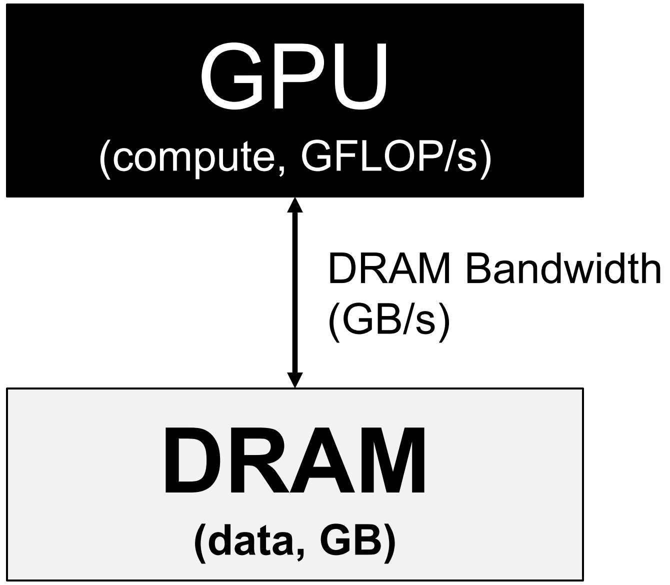


$$\text{GFLOP/s} = \min \begin{cases} \text{Peak GFLOP/s} \\ \text{AI * Peak GB/s} \end{cases}$$

Note, Arithmetic Intensity (AI) = FLOPs / Bytes (as presented to DRAM)

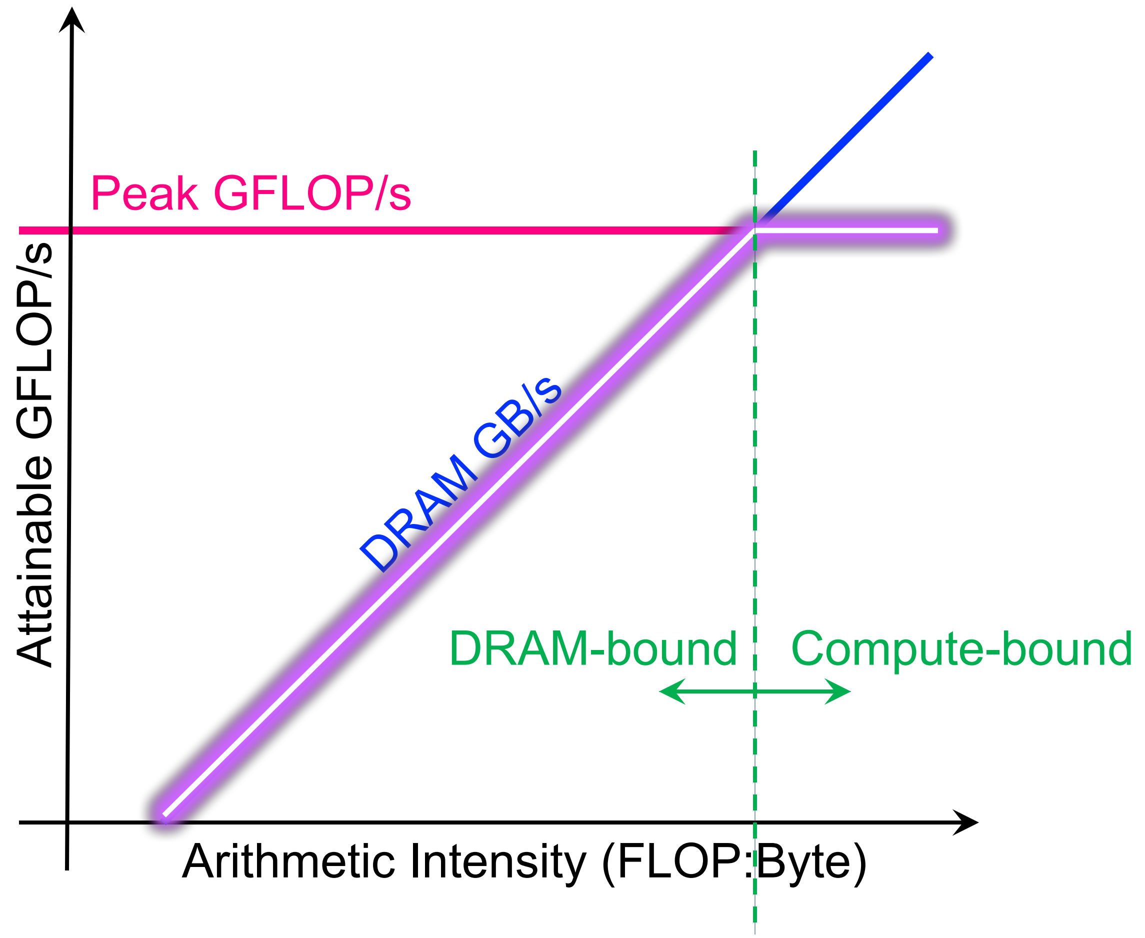
Arithmetic Intensity

- **Arithmetic Intensity** is the most important concept in Roofline.

- Measure of data locality (data reuse)
- Ratio of **Total FLOPs** performed to **Total Bytes** moved
- For the DRAM Roofline…
  - Total Bytes to/from DRAM and includes all cache and prefetcher effects
  - Can be **very different from total loads/stores** (bytes requested) due to cache reuse

(DRAM) Roofline

- Plot Roofline bound using Arithmetic Intensity as the x-axis
- **Log-log scale** makes it easy to doodle, extrapolate performance along Moore's Law, etc…
- Kernels with AI less than machine balance are ultimately DRAM bound (we'll refine this later…)



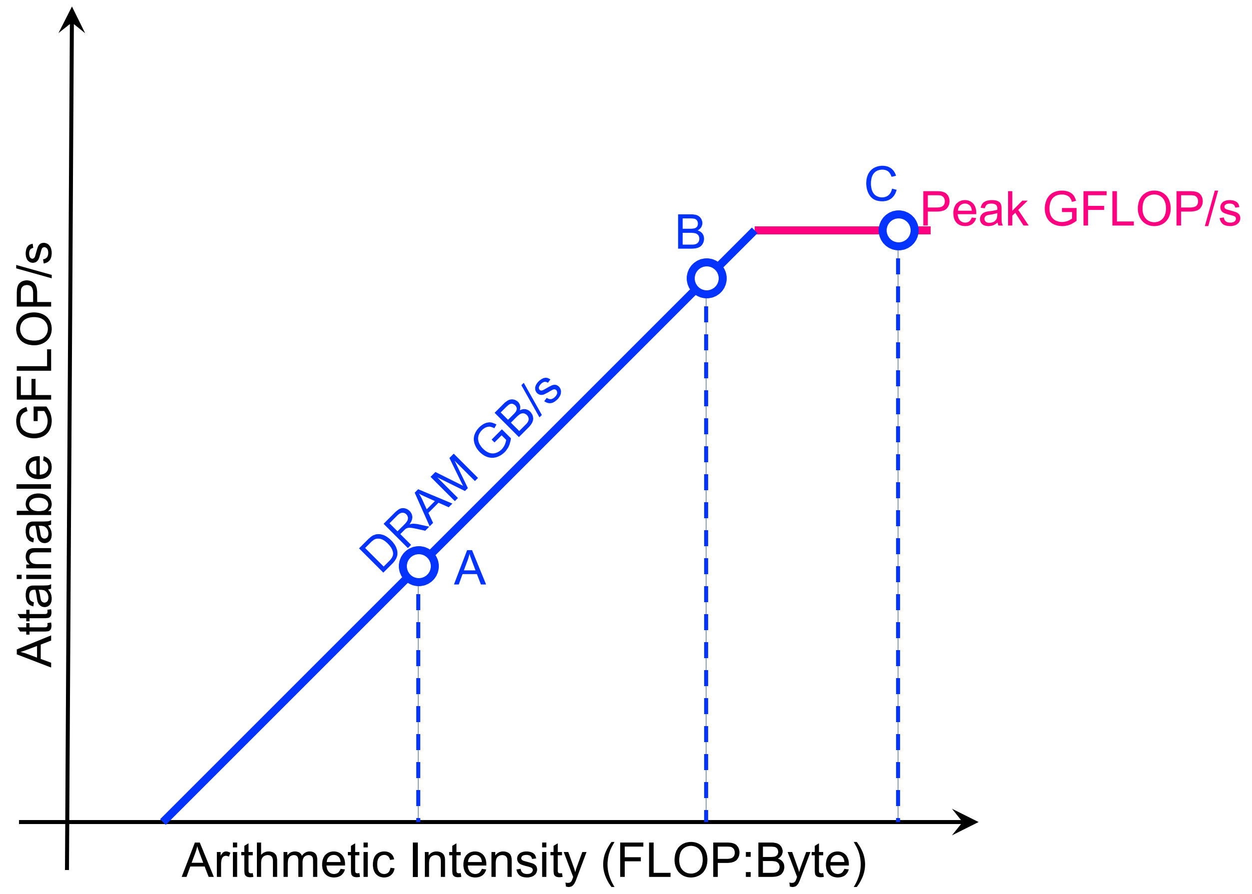
Transition @ AI ==
Peak Gflop/s / Peak GB/s ==
'Machine Balance'

Example

- Consider 3 kernels (A,B,C)
  - calculate or measure the **Arithmetic Intensity** for each
  - Determine the Roofline intercept for each kernel

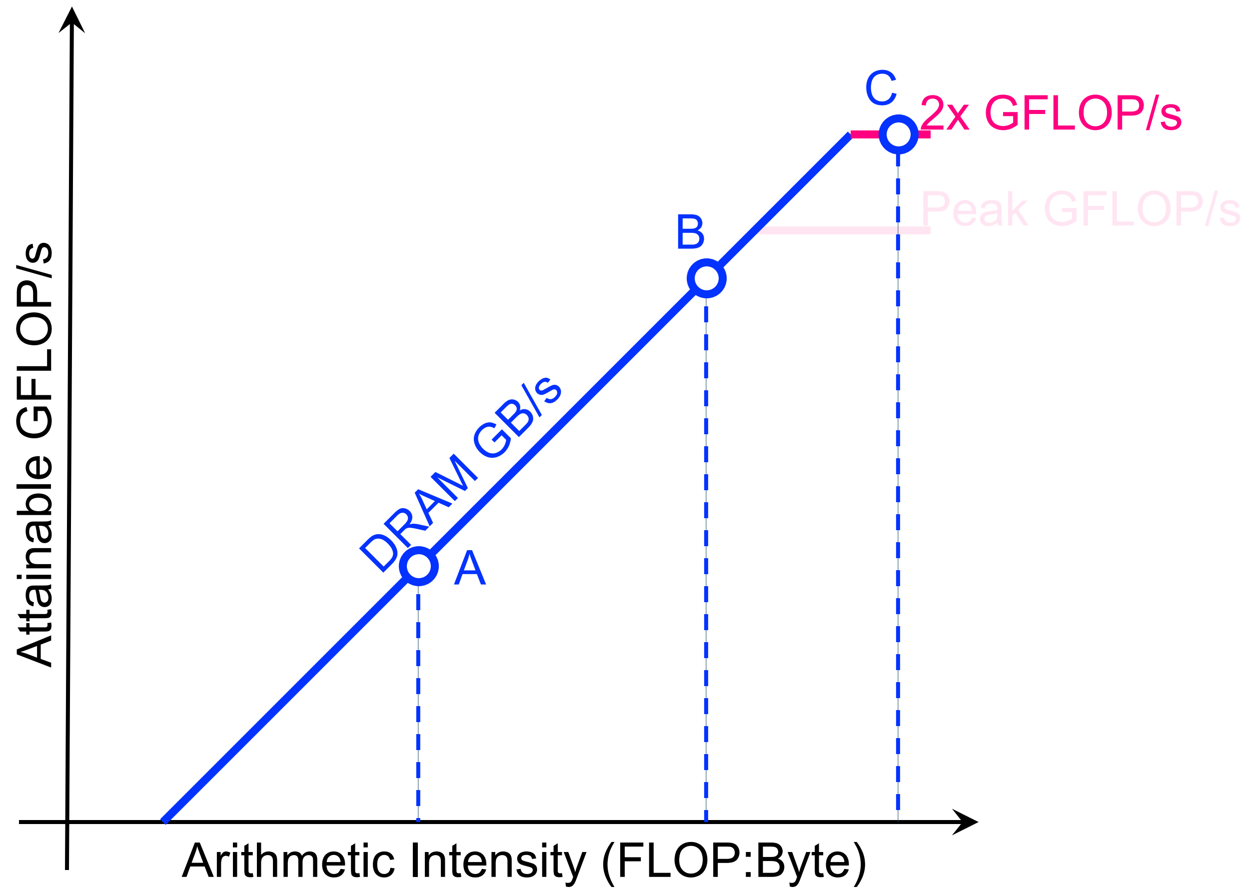
➢ **kernels A and B are bound by memory bandwidth**

➢ **kernel C is bound by peak FLOP/s**



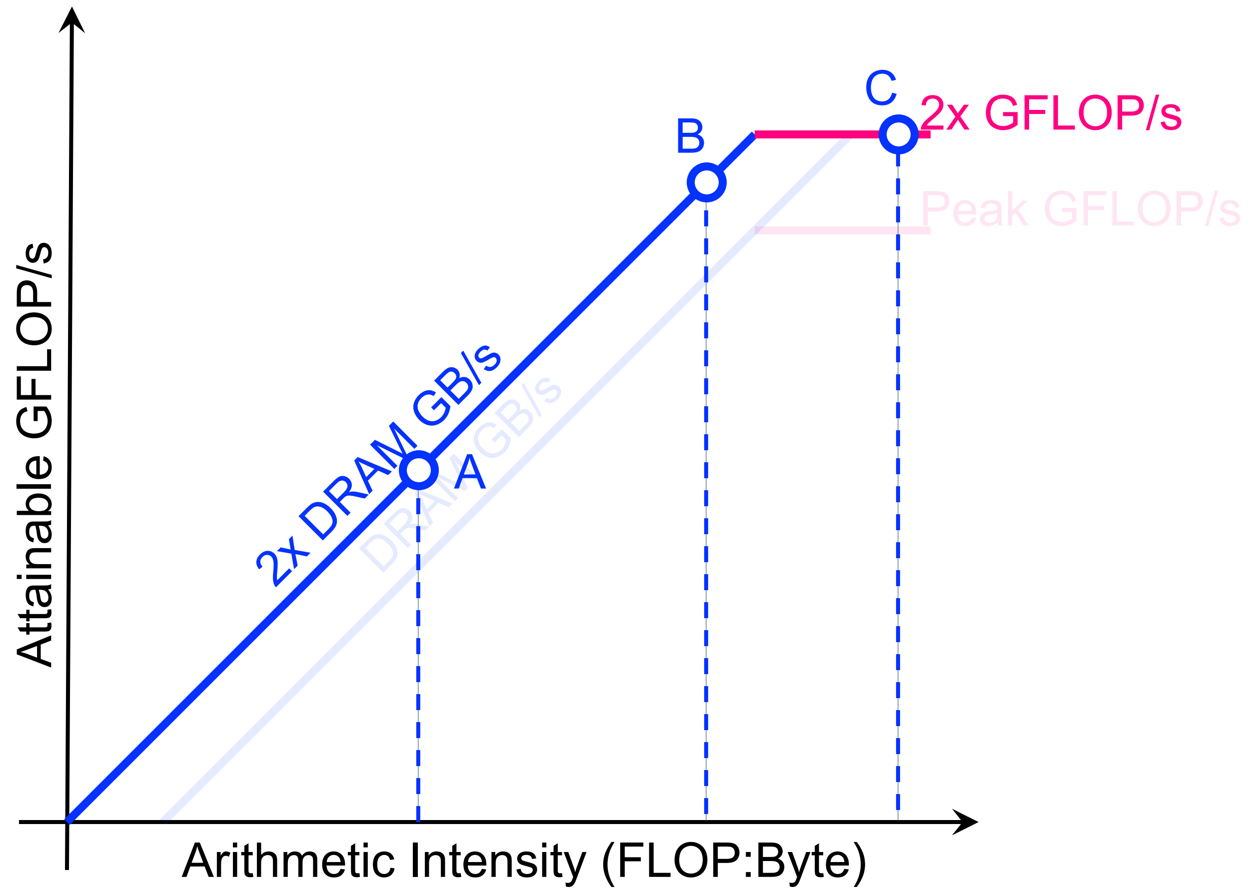
Scaling to Future GPUs

- Imagine you run on a future GPU with twice the peak FLOPs…
  - ➢ **kernel C's performance could double**
  - ✘ **kernels A and B will be no faster**



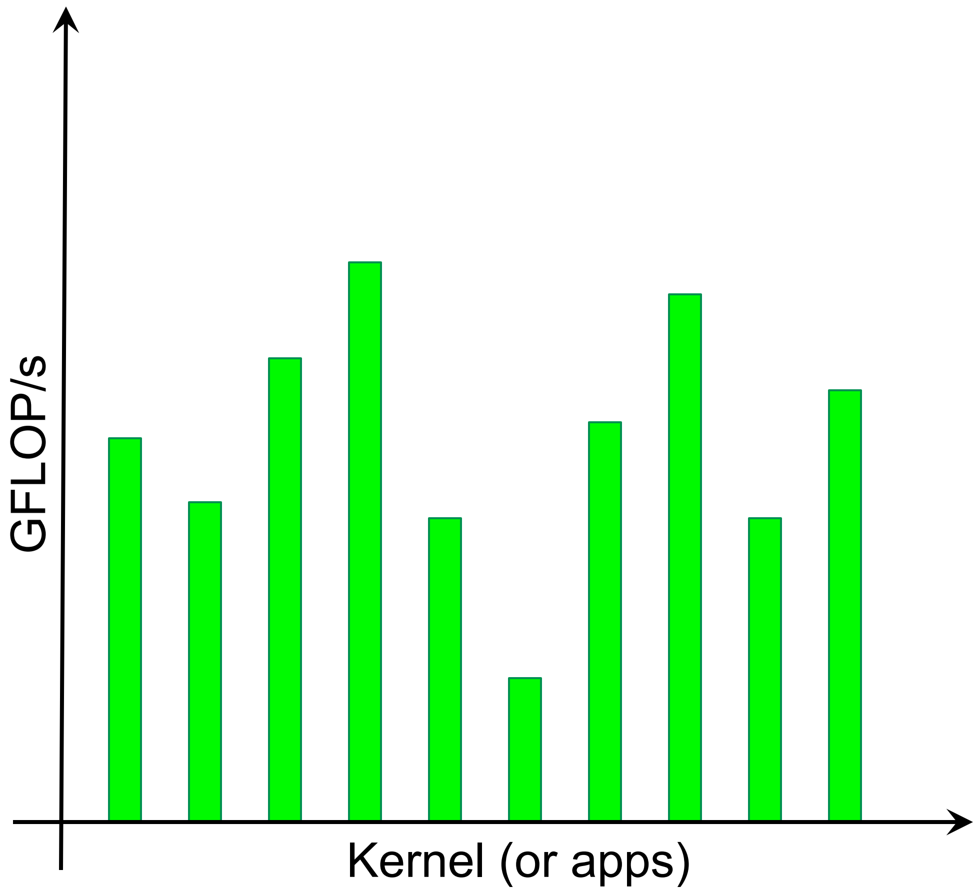
Scaling to Future GPUs

- What if that future GPU also doubled its memory bandwidth…
  - **kernel A and B's performance could also double**



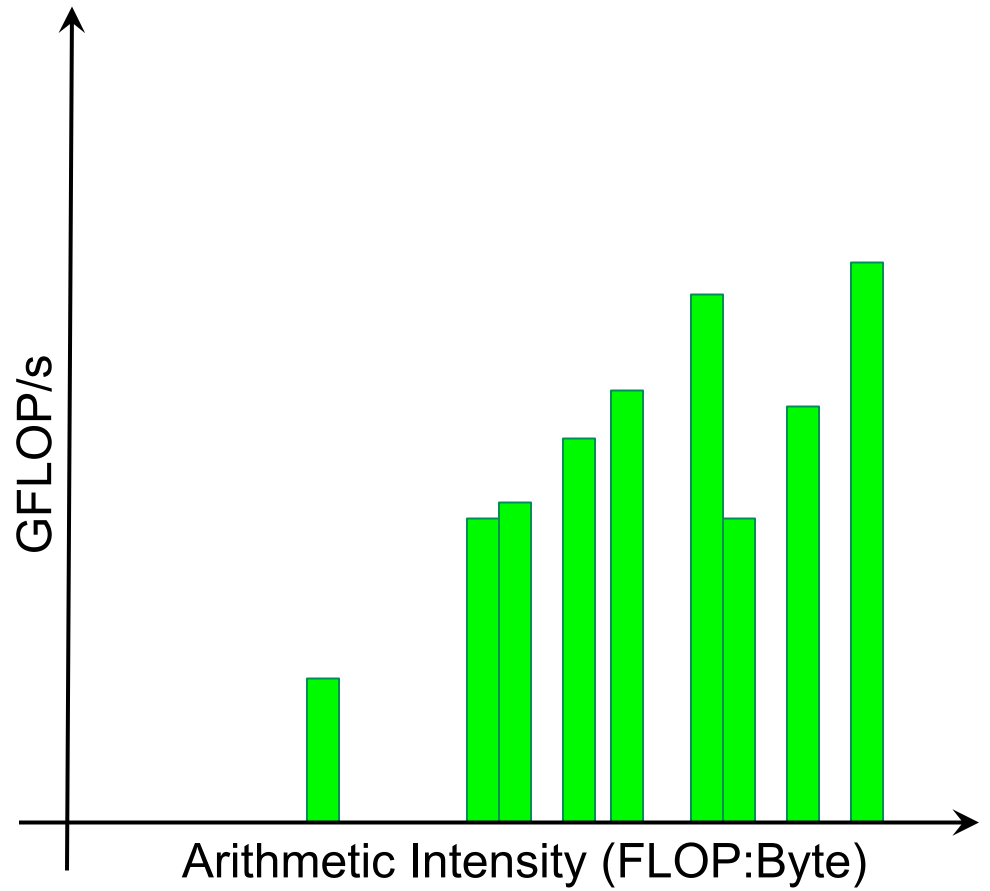
Why is Roofline Useful?

- Think back to our mix of loop nests where GFLOP/s alone wasn't useful…



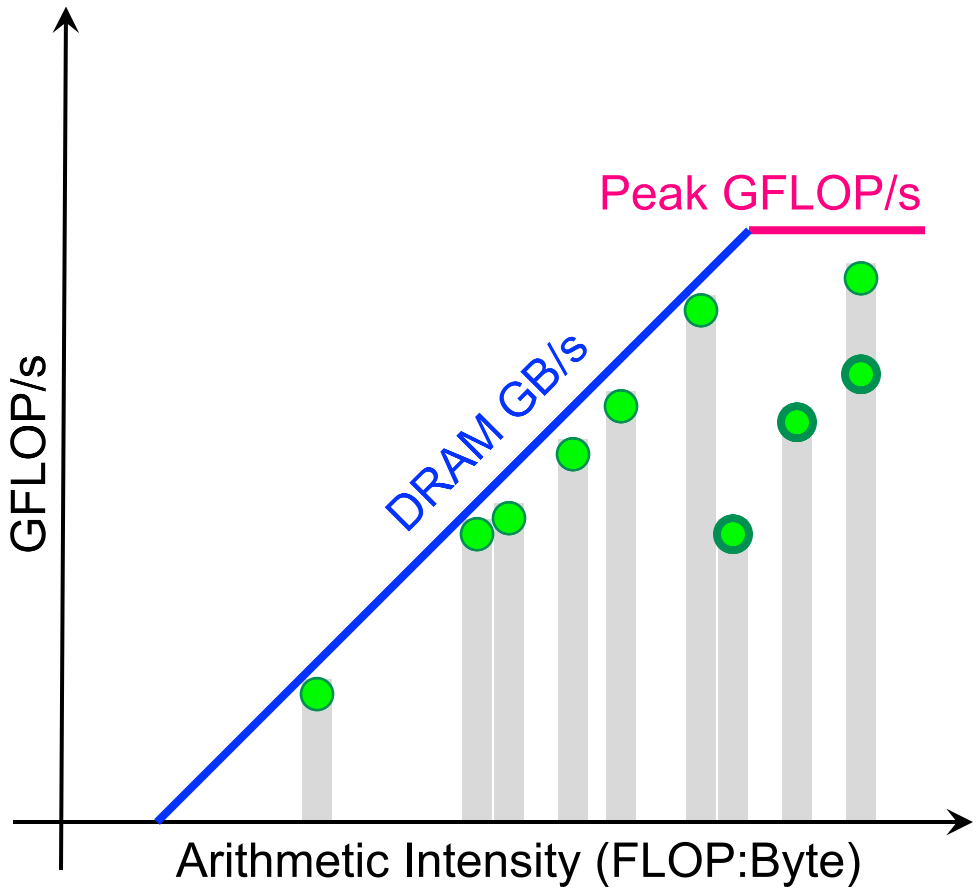
Why is Roofline Useful?

- We can sort kernels by AI …



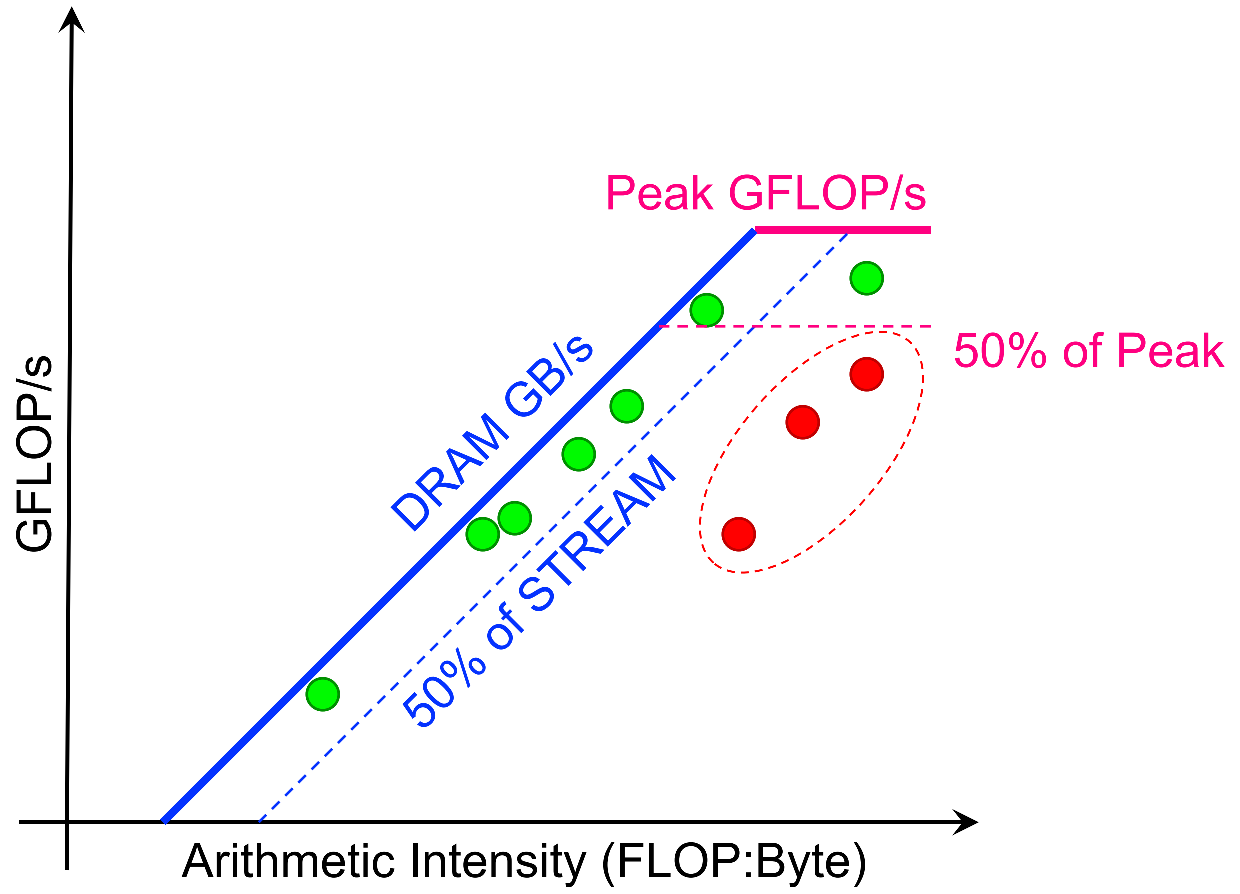
Why is Roofline Useful?

- We can sort kernels by AI …
- … and compare performance relative to machine capabilities



Why is Roofline Useful?

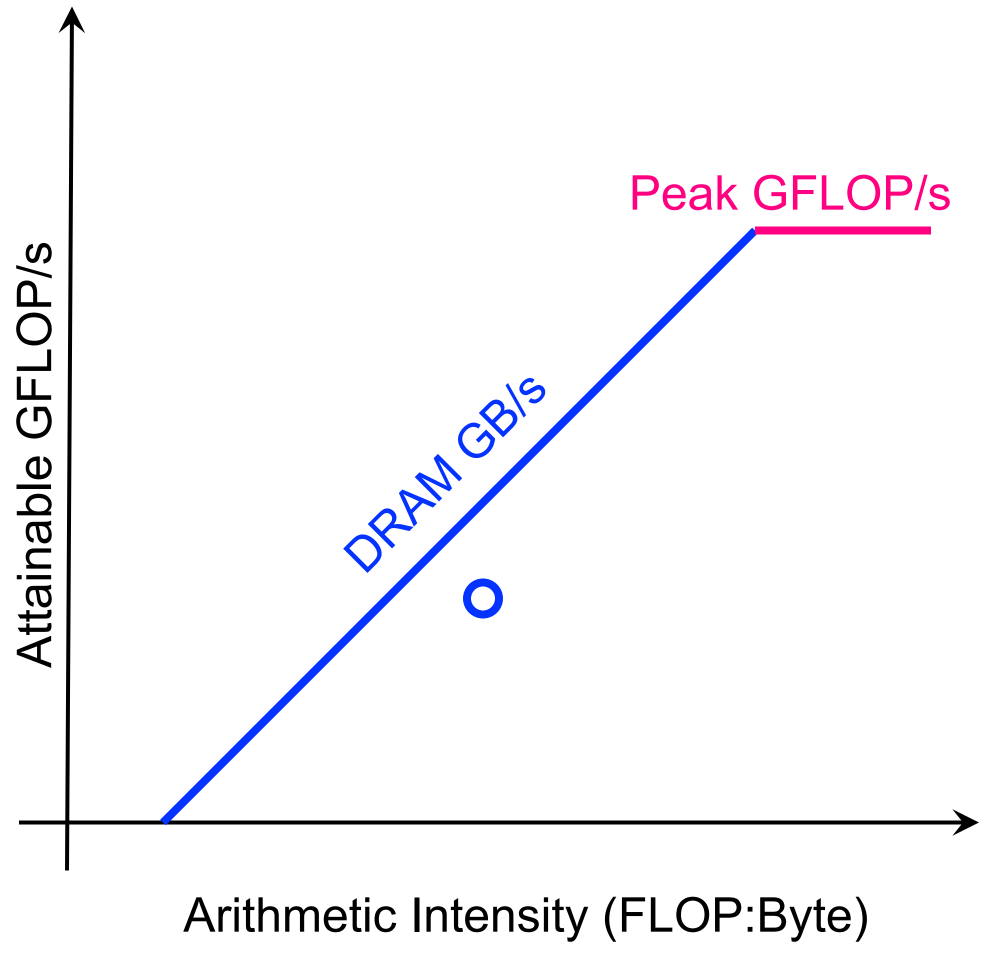
- Kernels near the roofline are making good use of computational resources…
  - **kernels can have low performance (GFLOP/s), but make good use of a machine**
  - **kernels can have high performance (GFLOP/s), but make poor use of a machine**



Can Performance Be Below Roofline?

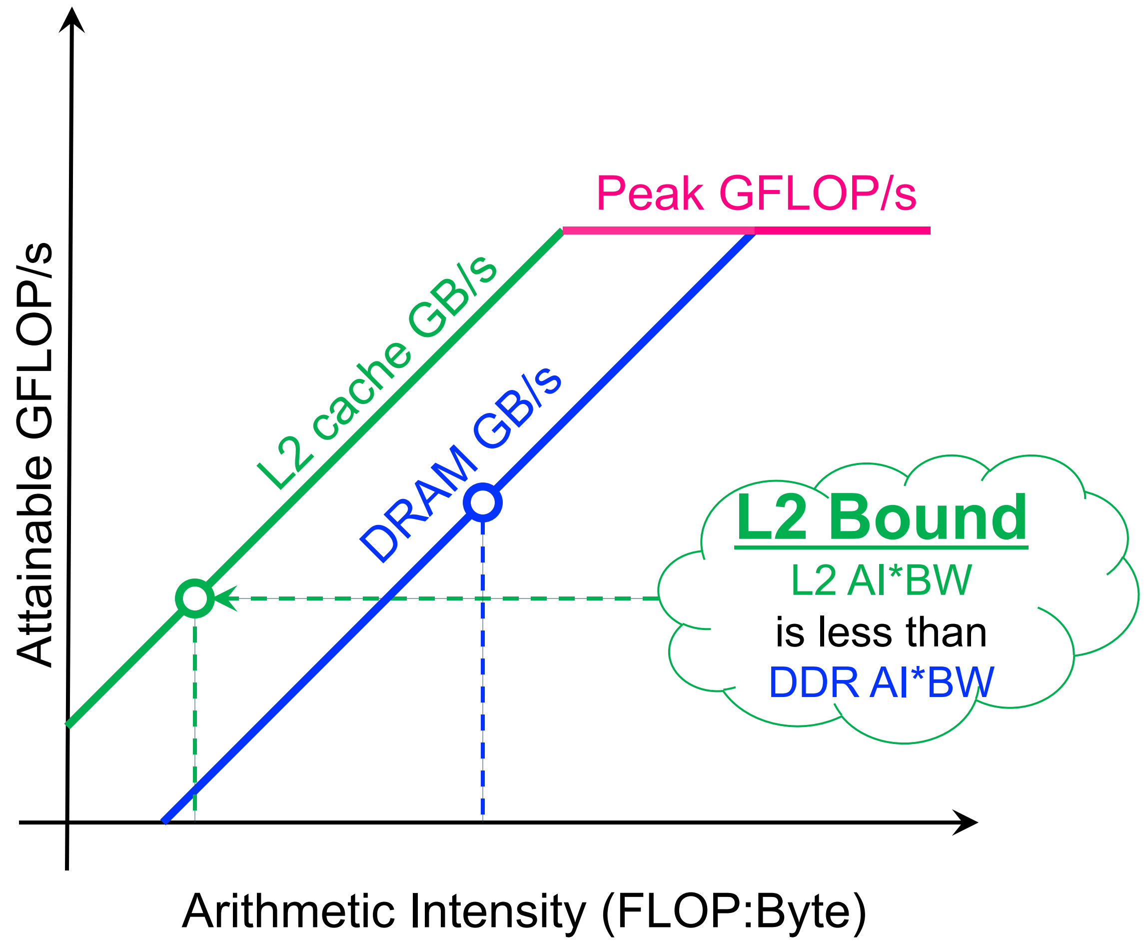
- Analogous to asking whether one can always attain either…
  - Peak Bandwidth
  - Peak GFLOP/s

- **Sure, there can be other performance bottlenecks…**
  - Cache bandwidth / locality
  - Lack of FMA / tensor instructions
  - Thread divergence / predication
  - Too many non-FP instructions
  - …



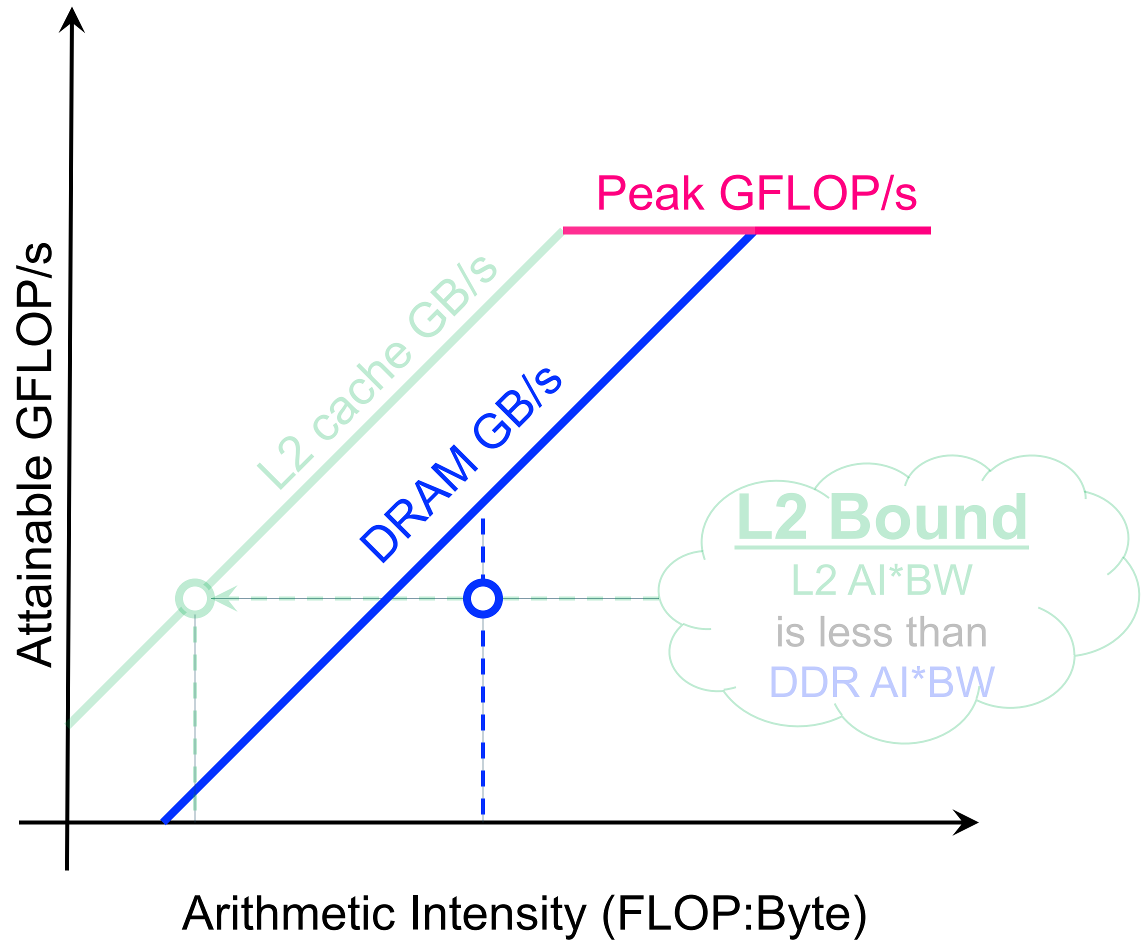
Cache Effects…

- Hierarchical Roofline Model
- Construct superposition of Rooflines…
  - Measure AI and bandwidth for each level of memory/cache
  - Loop nests will have multiple AI's and multiple performance bounds…
  - **… but performance is ultimately the minimum of these bounds.**



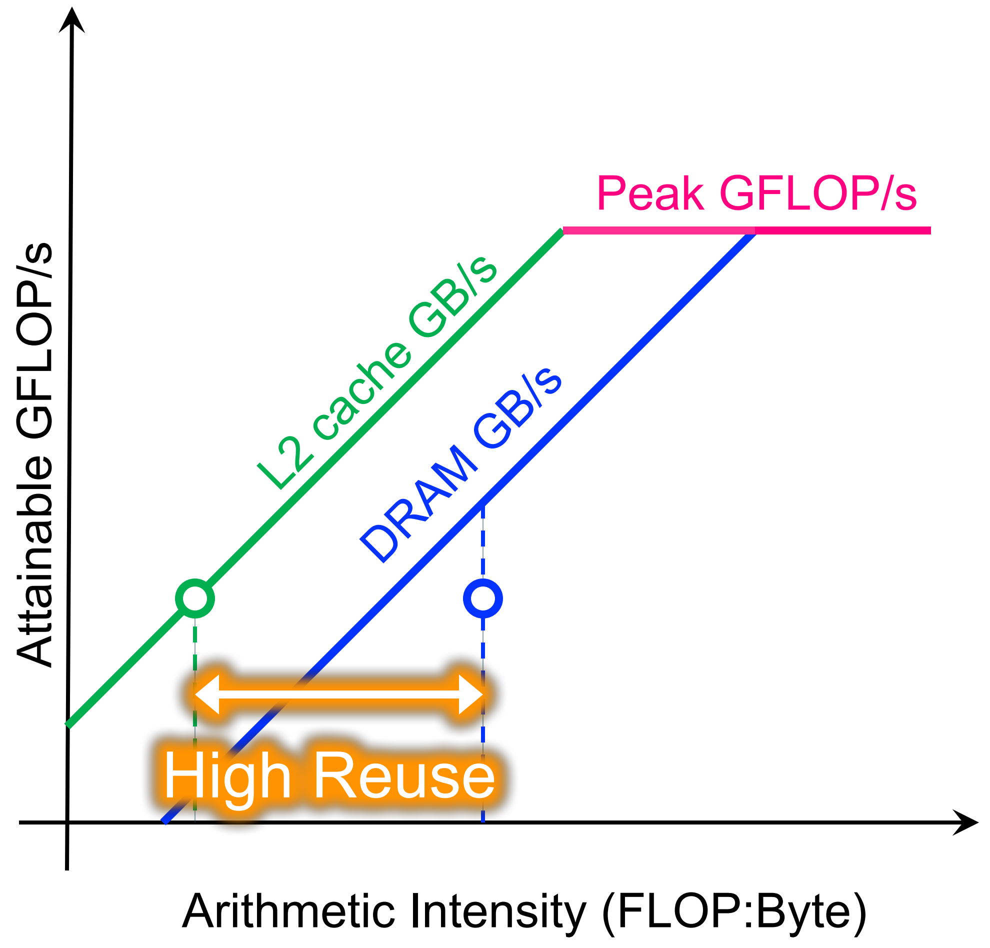
Cache Effects…

- Hierarchical Roofline Model
- Construct superposition of Rooflines…
  - Measure AI and bandwidth for each level of memory/cache
  - Loop nests will have multiple AI's and multiple performance bounds…
  - **… but performance is ultimately the minimum of these bounds.**
- Extend to other memories…
  - L1 / Shared
  - System



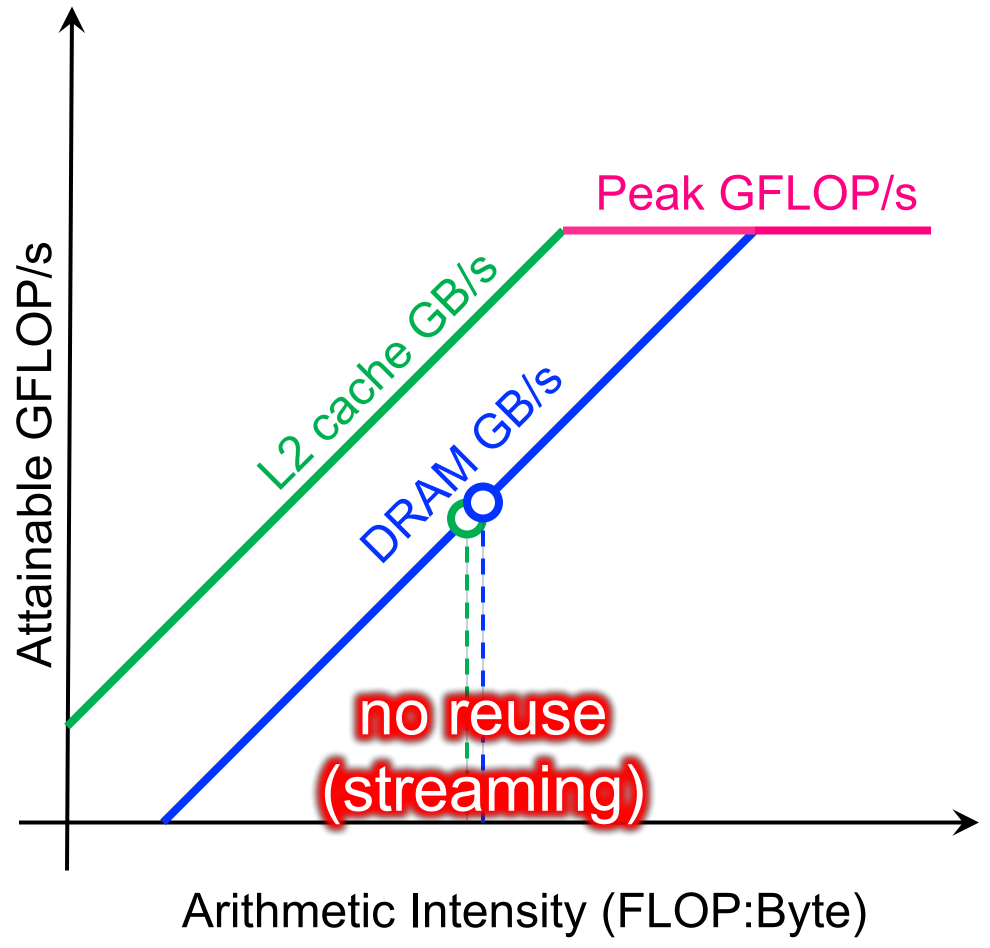
Insights – Exploiting Caches

- Widely separated Arithmetic Intensities indicate high reuse in the cache



Insights – Exploiting Caches

- Widely separated Arithmetic Intensities indicate high reuse in the cache
- Similar Arithmetic Intensities indicate effectively no cache reuse (**== streaming**)
- As one changes problem size, L2 and DRAM arithmetic intensities can behave very differently



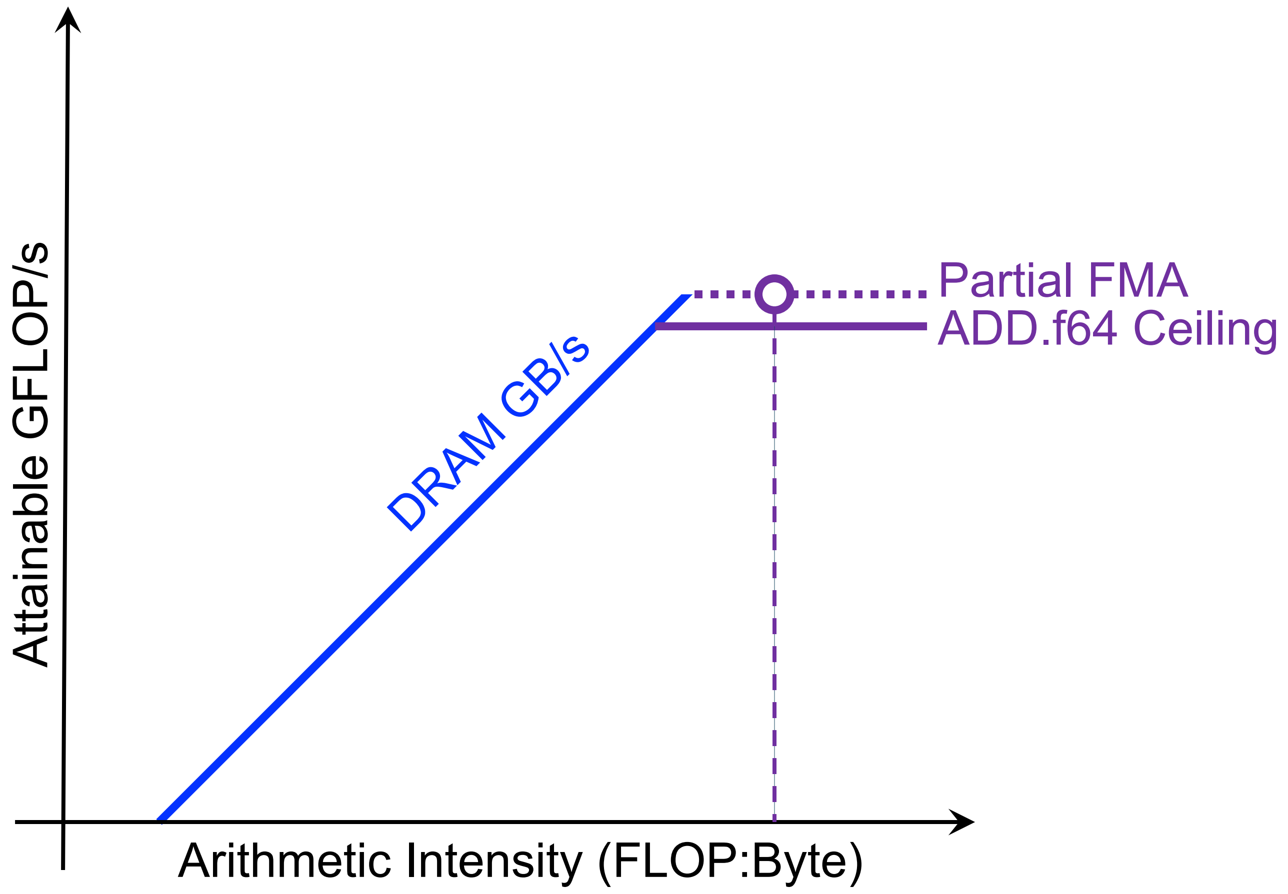
Failure to Exploit CISC Instructions

- Death of Moore's Law is motivating a return of Complex Instruction Set Computing (CISC)
- Modern CPUs and GPUs are increasingly reliant on special (fused) instructions that perform multiple operations.
  - FMA (Fused Multiply Add): z=a*x+y …*z,x,y are vectors or scalars*
  - 4FMA (quad FMA): z=A*x+z …*A is a FP32 matrix; x,z are vectors*
  - HMMA (Tensor Core): Z=AB+C …*Z,A,B,C are FP16 matrices*
  - …

➢ **Performance is now a weighted average of Mul/Add, FMA, and HMMA operations.**

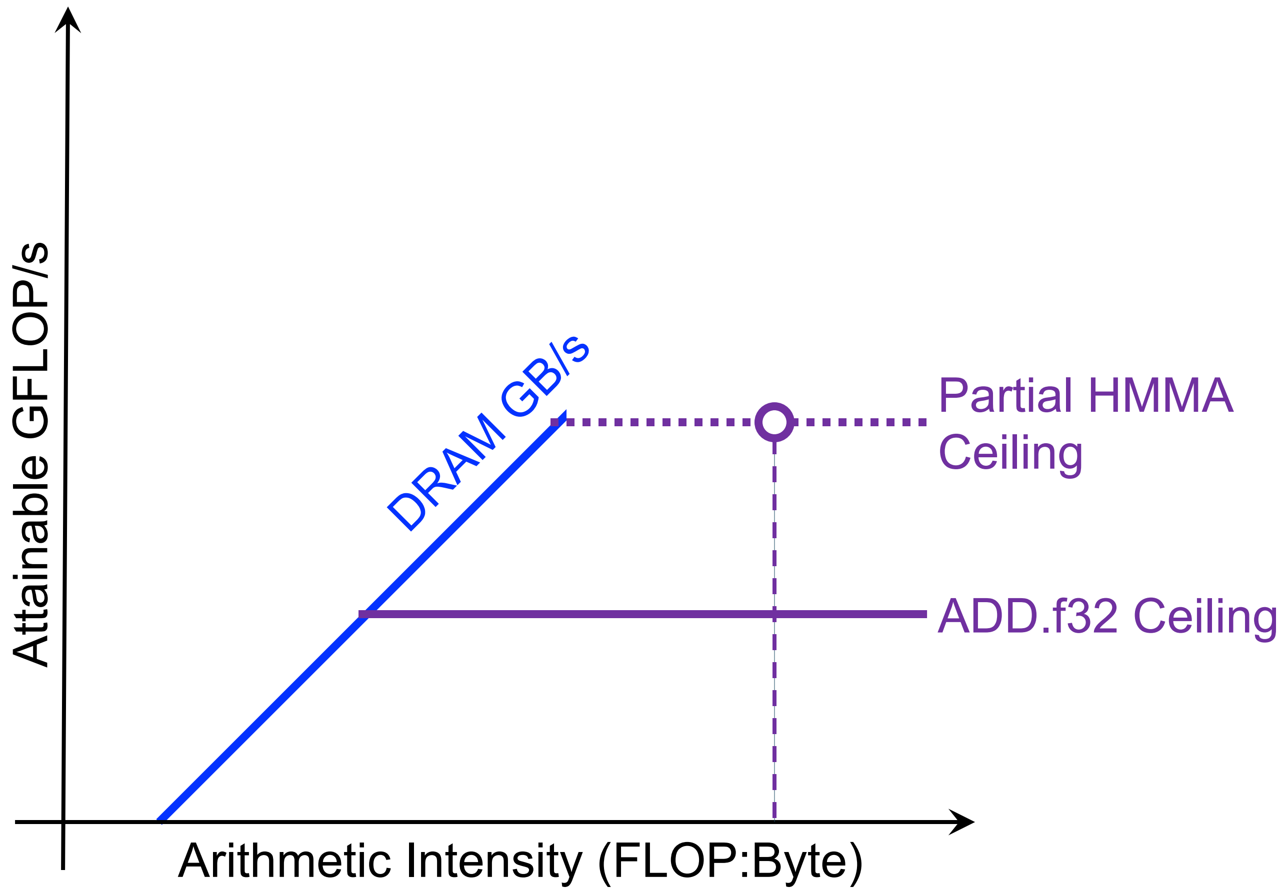
Failure to Exploit CISC Instructions

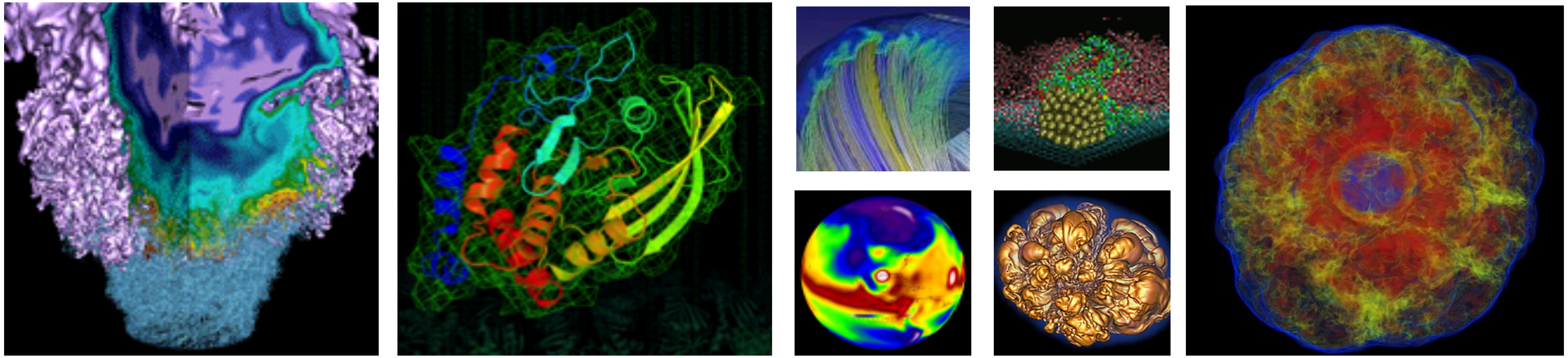
- Total lack of FMA reduces Volta performance by 2x…
  - **creates ADD.f64 <u>ceiling</u>**
- In reality, applications are a mix of FMA.f64, ADD.f64, and MUL.f64…
  - Performance is a weighted average
  - ➢ **Produces a partial FMA ceiling that bounds kernel performance**



Failure to Exploit CISC Instructions

- On Volta, Tensor cores provide **125 TFLOPs** of FP16 performance (vs. 15 for FP32)
- However, kernels/apps will mix HMMA with FMA, MULs, ADDs, …
  - **A few non-HMMA operations can quickly limit Tensor core performance**





Using Roofline To Drive Optimization

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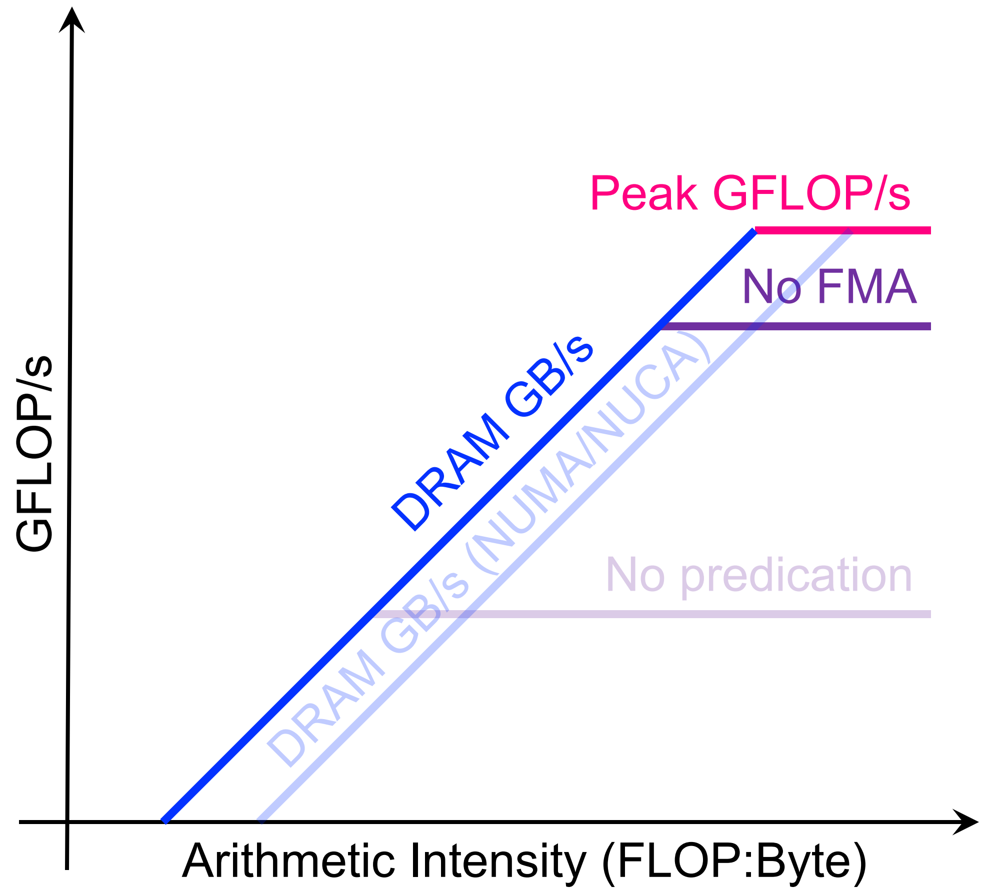
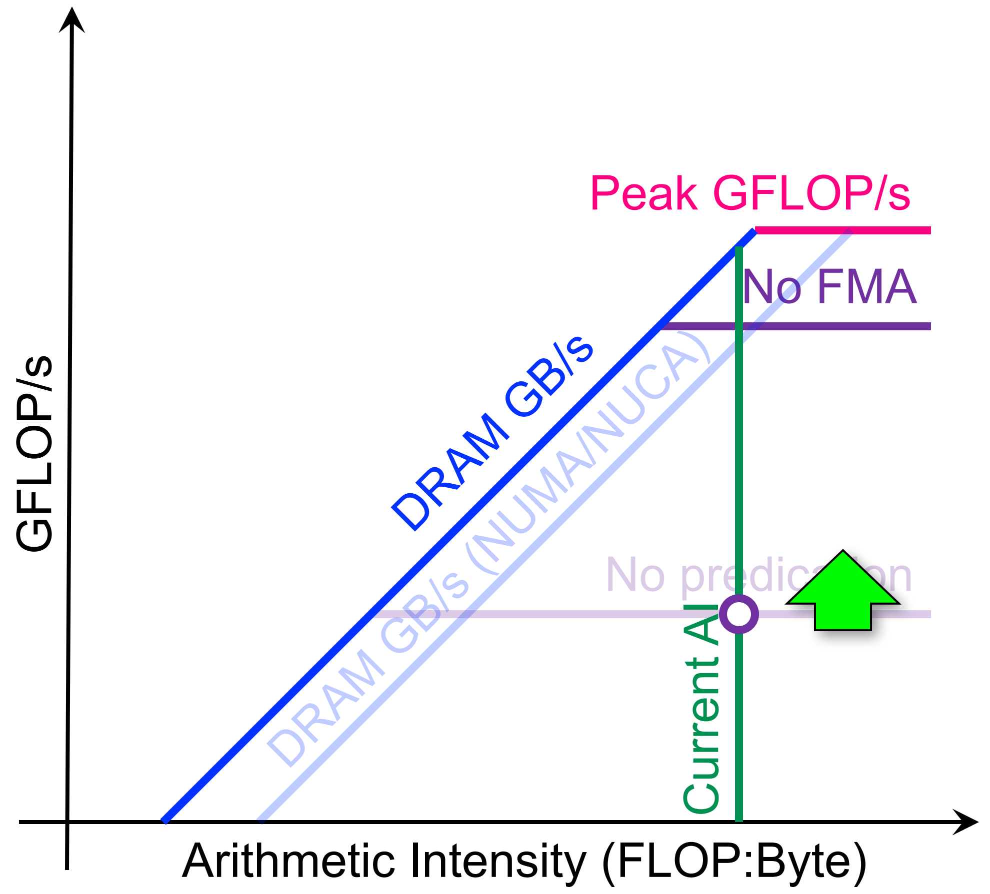
Driving Performance Optimization

- Broadly speaking, there are three approaches to improving performance:



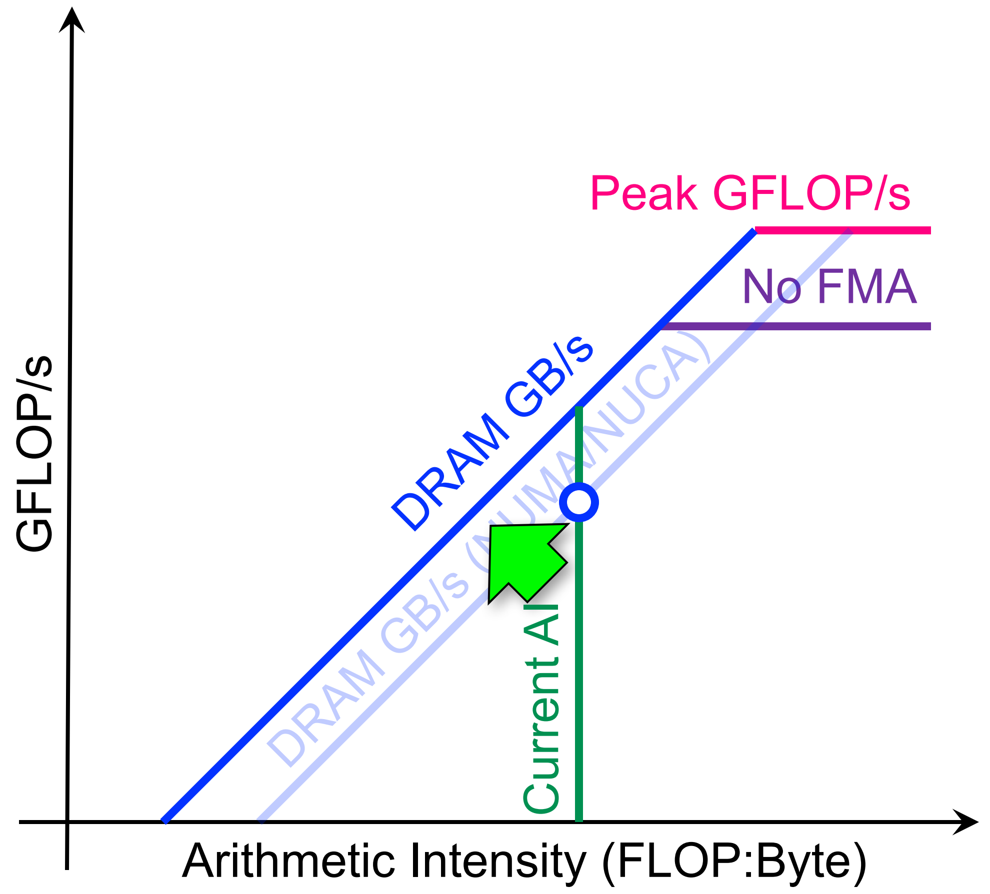
Driving Performance Optimization

- Broadly speaking, there are three approaches to improving performance:
- **Maximize SM performance (e.g. minimize predication)**



Driving Performance Optimization

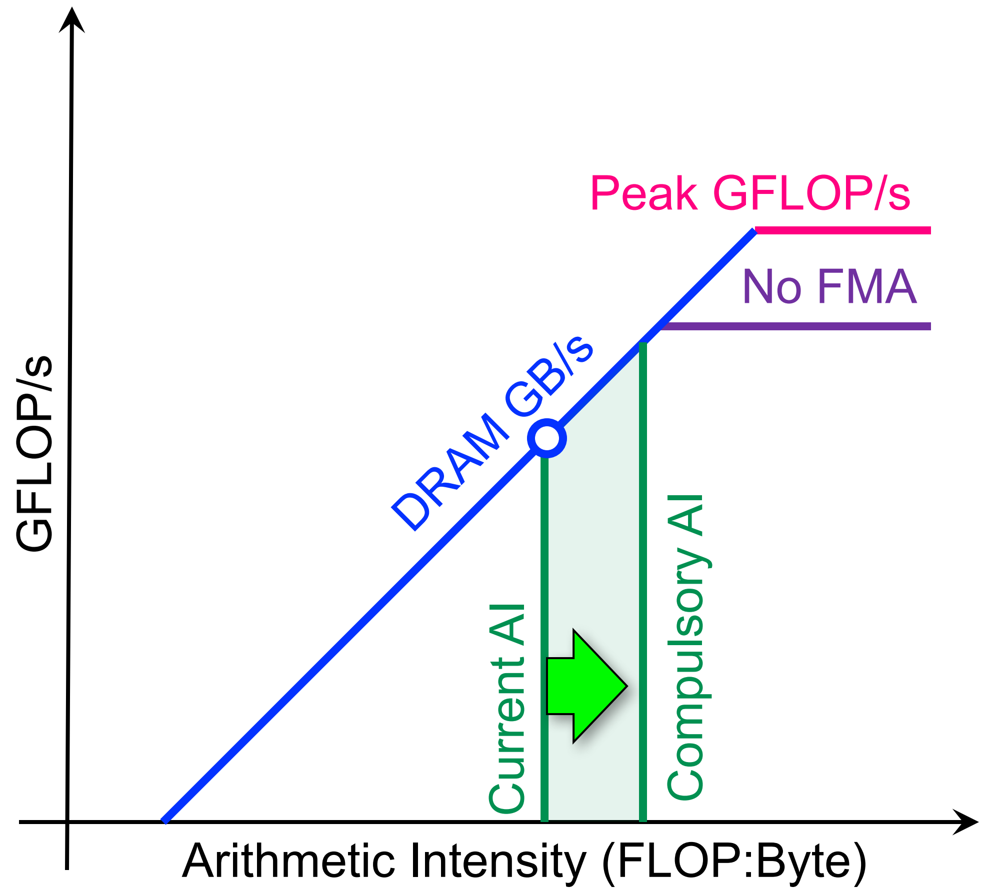
- Broadly speaking, there are three approaches to improving performance:
- Maximize SM performance (e.g. minimize predication)
- **Maximize memory bandwidth (e.g. avoid pathological memory access patterns)**

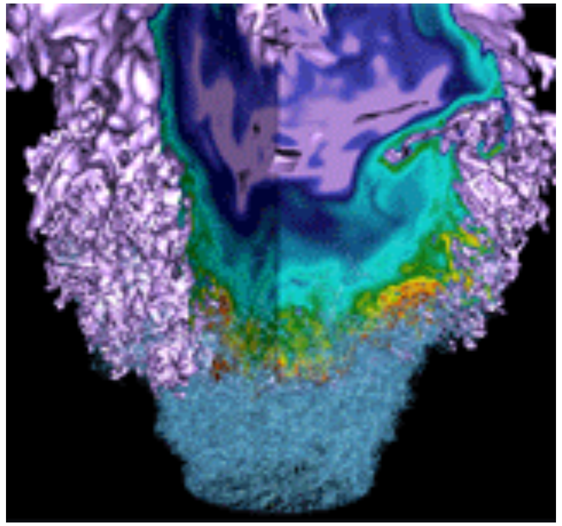
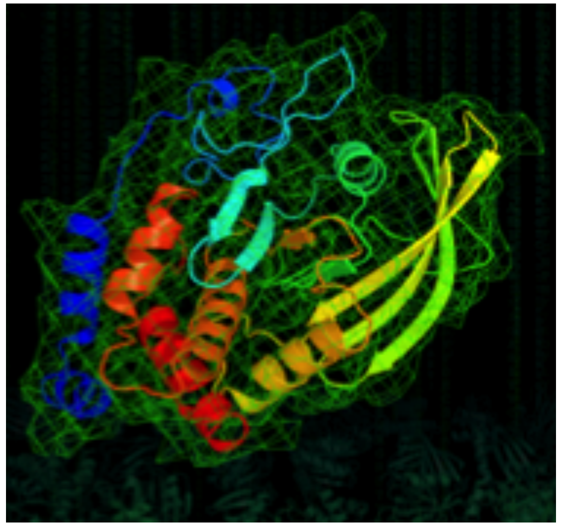
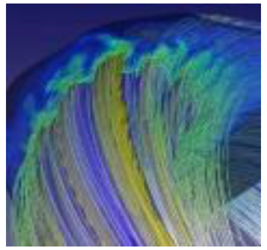
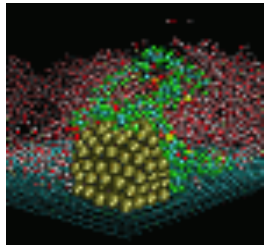
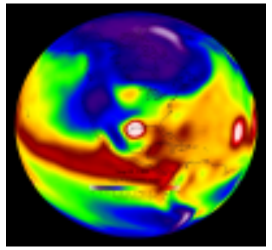
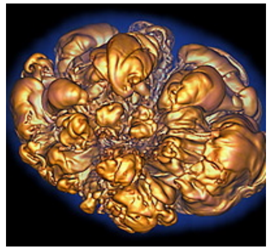
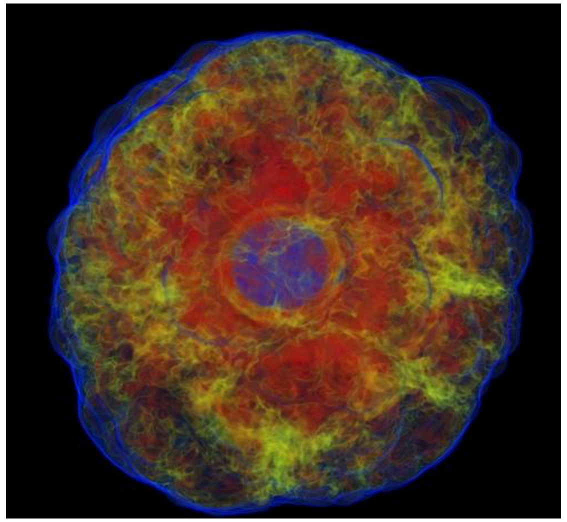


Driving Performance Optimization

- Broadly speaking, there are three approaches to improving performance:
- Maximize SM performance (e.g. minimize predication)
- Maximize memory bandwidth (e.g. avoid pathological memory access patterns)
- **Minimize data movement**

  (i.e. exploit reuse)



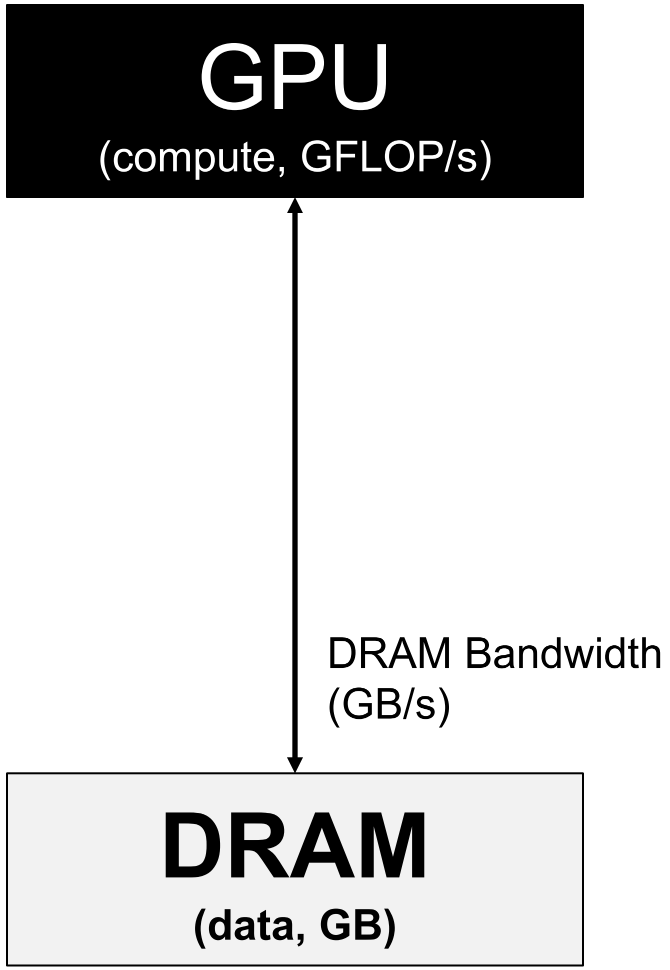








Estimating Arithmetic Intensity

DRAM vs L1 Arithmetic Intensity

- Consider a 7-point constant coefficient stencil…
  - 7 FLOPs
  - 8 memory references (7 reads, 1 store) per point
  - **AI = 0.11 FLOPs per byte (L1)**

```
#pragma omp parallel for
for(k=1;k<dim+1;k++){
for(j=1;j<dim+1;j++){
for(i=1;i<dim+1;i++){
  new[k][j][i] = -6.0*old[k  ][j  ][i  ]
                    + old[k  ][j  ][i-1]
                    + old[k  ][j  ][i+1]
                    + old[k  ][j-1][i  ]
                    + old[k  ][j+1][i  ]
                    + old[k-1][j  ][i  ]
                    + old[k+1][j  ][i  ];
}}}
```



DRAM vs L1 Arithmetic Intensity

- Consider a 7-point constant coefficient stencil…
  - 7 FLOPs
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  - Cache can filter all but 1 read and 1 write per point
  - **AI = 0.44 FLOPs per byte**

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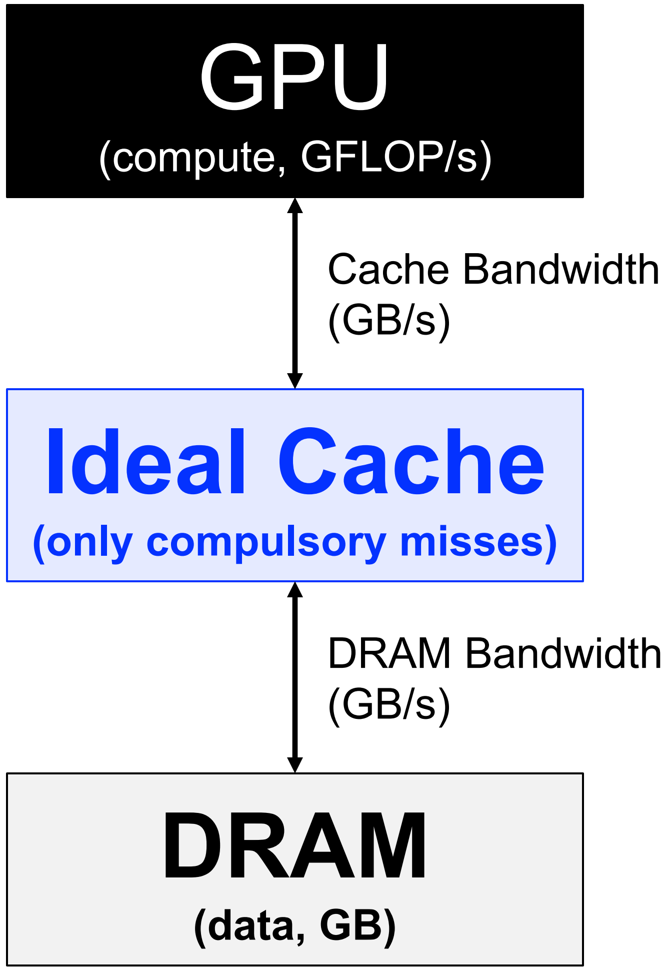

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#pragma omp parallel for
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for(i=1;i<dim+1;i++){
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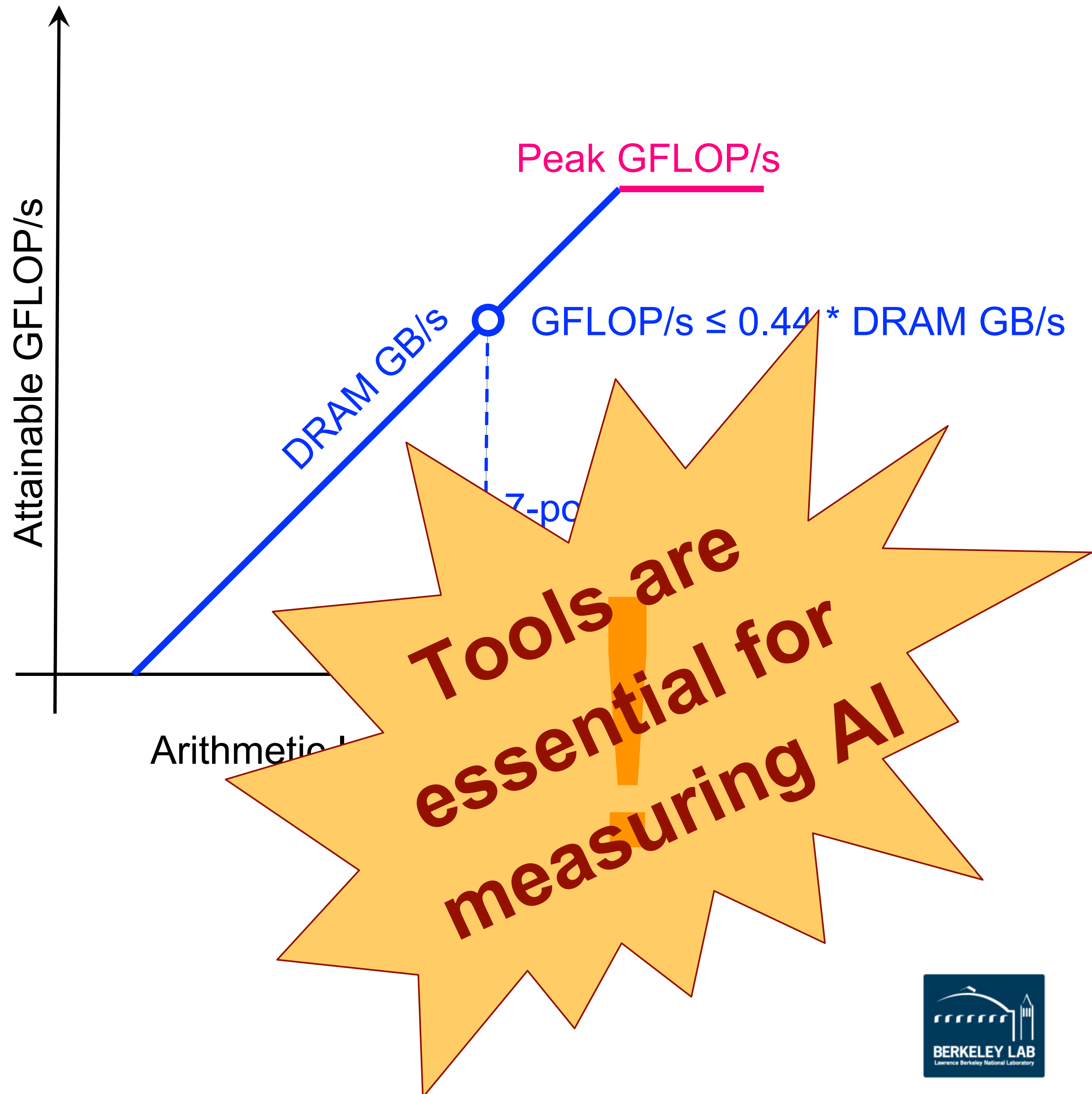

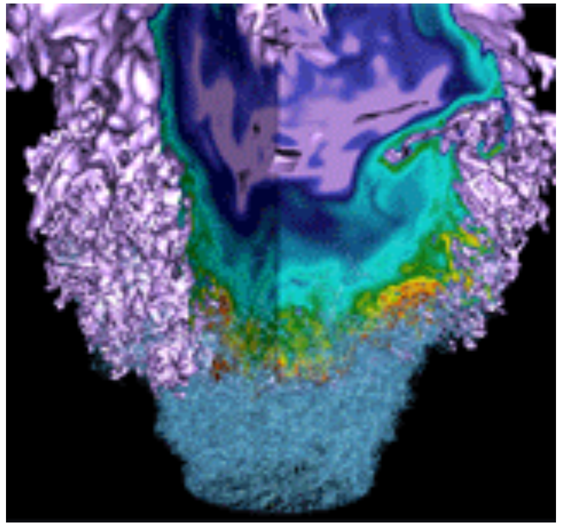
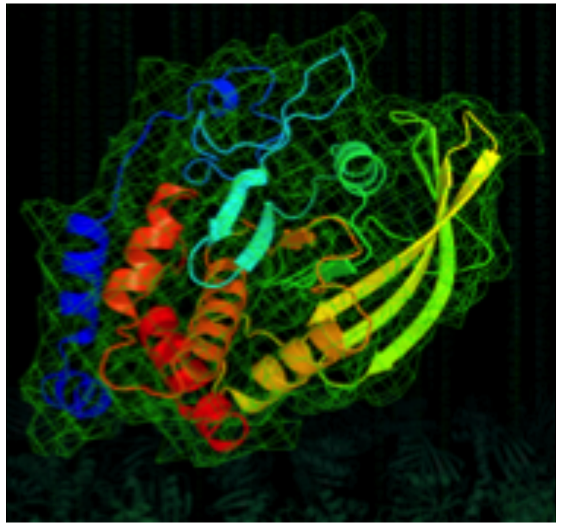
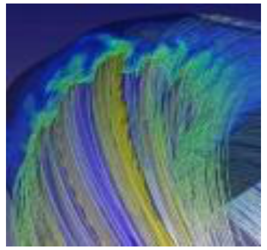
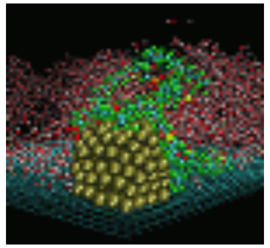
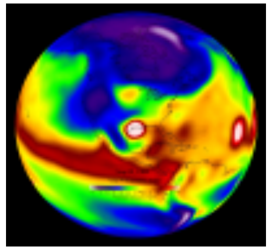
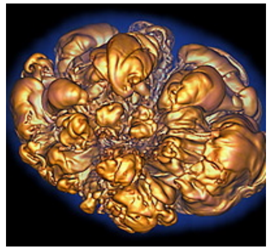
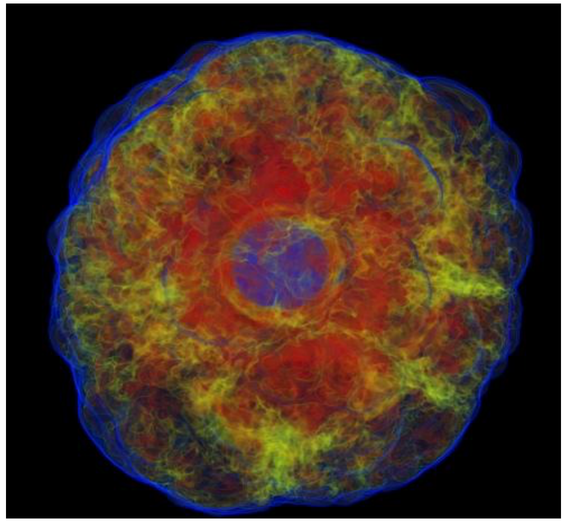

Collecting Roofline Data with nvprof

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General Roofline Data Collection

Most kernels are more complicated than the 7-point stencil…

General Roofline Data Collection

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How do we measure the total number of FLOPs?
How do we measure the total number of bytes moved (read/write, L1/L2/HBM)?
How do we measure the runtime for each kernel?

How do we know the peak bandwidth (L1/L2/HBM) and the peak FLOP/s for the architecture?

General Roofline Data Collection

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nvprof

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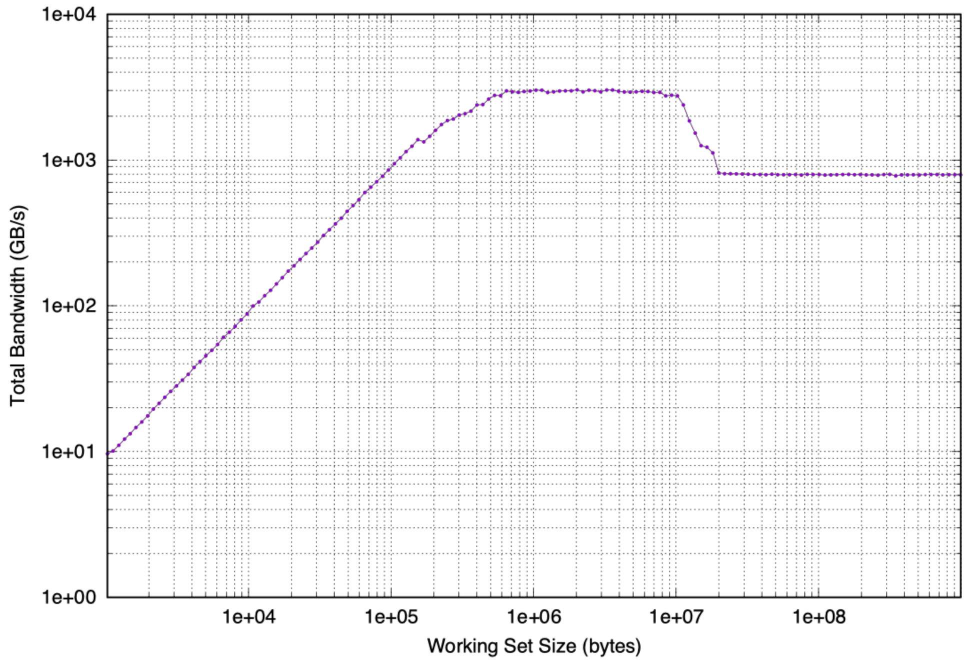
ERT

Step 1. Collect Roofline Ceilings

- **Empirical Roofline Toolkit (ERT)**
  - Different than the architecture specs, **MORE REALISTIC**
  - Reflects **actual** execution environment (power constraints, *etc*)
  - Sweeps through a range of configurations, and **statistically stable**
    - Data elements per thread
    - FLOPs per data element
    - Threadblocks/threads
    - Trails per dataset
    - *etc*



ERT Configuration

Kernel.c
- actual compute
- customizable

Driver.c
- setup
- call kernels
- loop over parameters

config script
- set up ranges of parameters

job script
- submit the job and run it

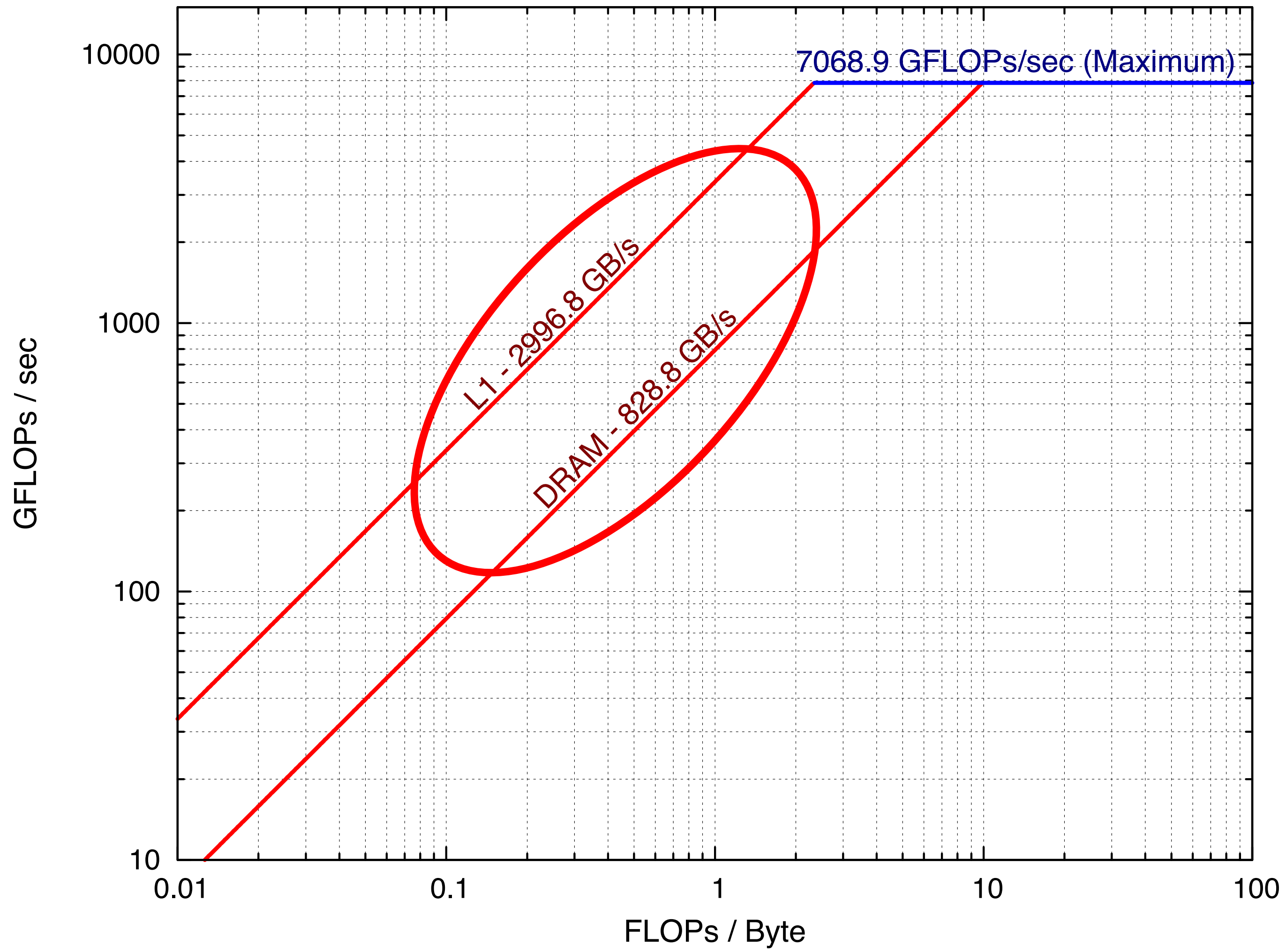
ERT Output

roofline.json


```
"gbytes": {
  "data": [
    [
      "L1",
      2996.82
    ],
    [
      "DRAM",
      828.83
    ]
  ],
```

```
"gflops": {
  "data": [
    [
      "GFLOPs",
      7068.90
    ]
  ],
```


roofline.ps



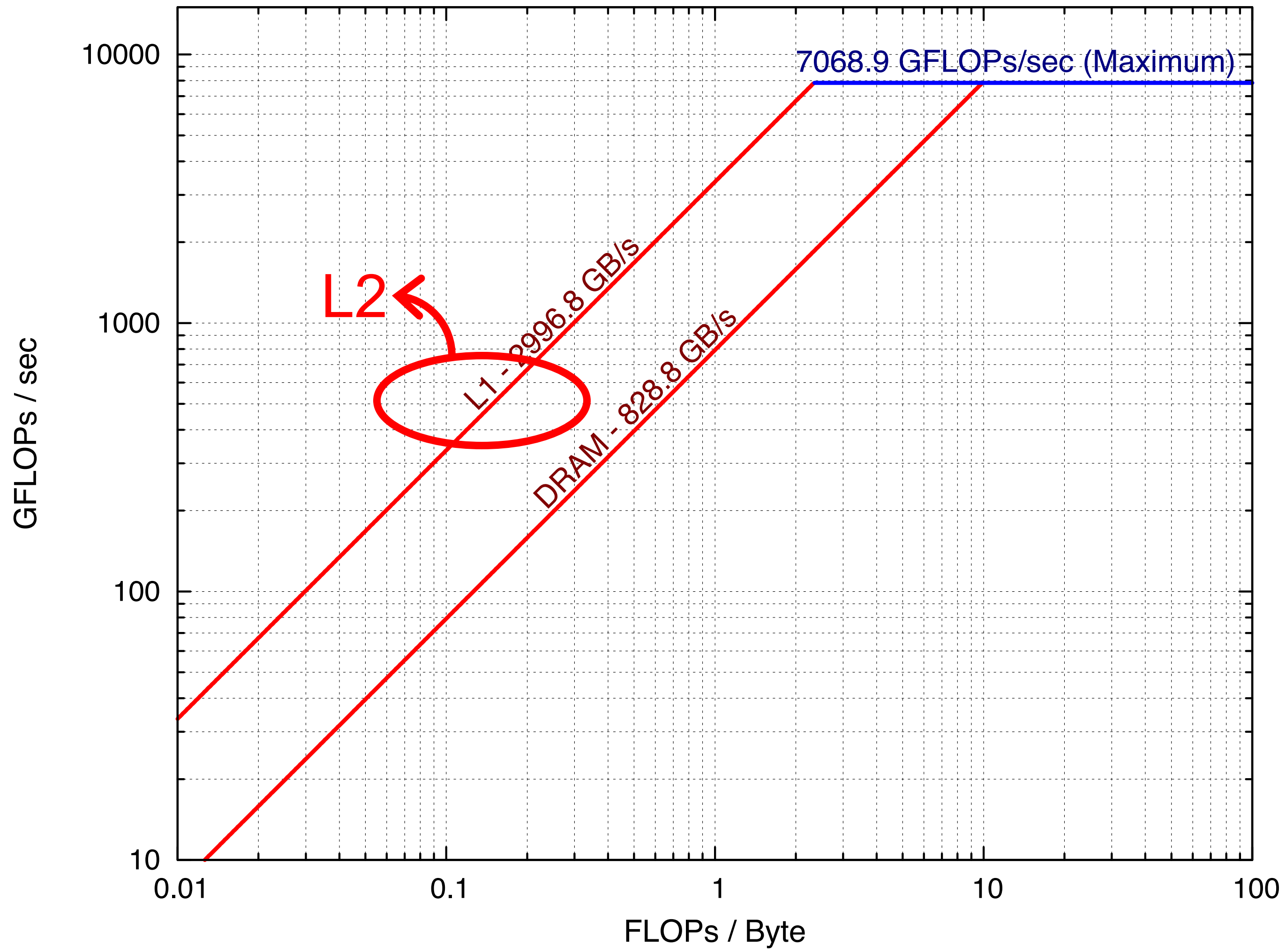
ERT Output

roofline.json


```
"gbytes": {
  "data": [
    [
      "L1",
      2996.82
    ],
    [
      "DRAM",
      828.83
    ]
  ],
```

```
"gflops": {
  "data": [
    [
      "GFLOPs",
      7068.90
    ]
  ],
```


roofline.ps



NVIDIA V100 -- Voltar at UOregon

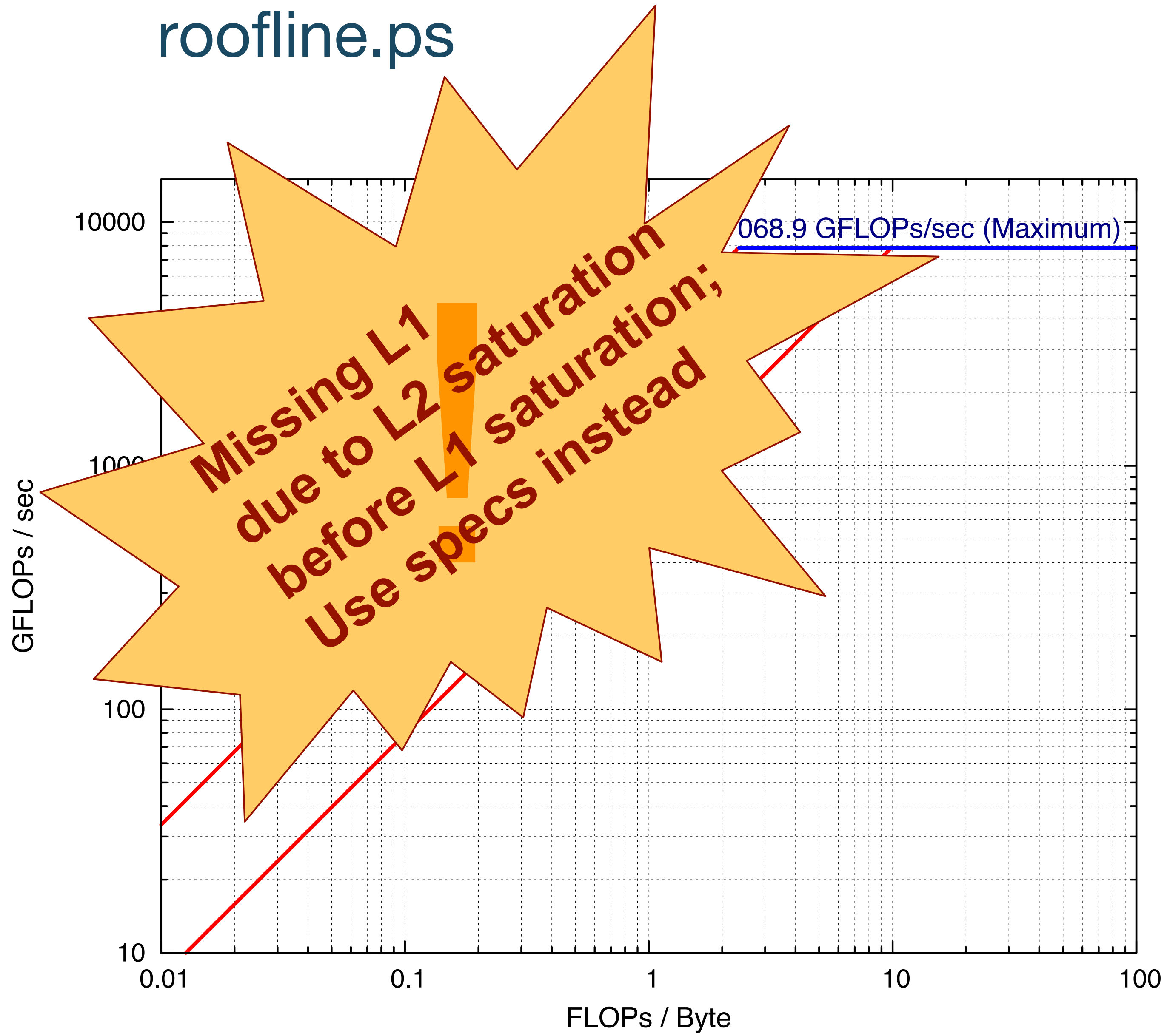
ERT Output

roofline.json

```
"gbytes": {
   "data": [
      [
         "L1",
         2996.82
      ],
      [
         "DRAM",
         828.83
      ]
   ],
```

```
"gflops": {
   "data": [
      [
         "GFLOPs",
         7068.90
      ]
   ],
```

roofline.ps



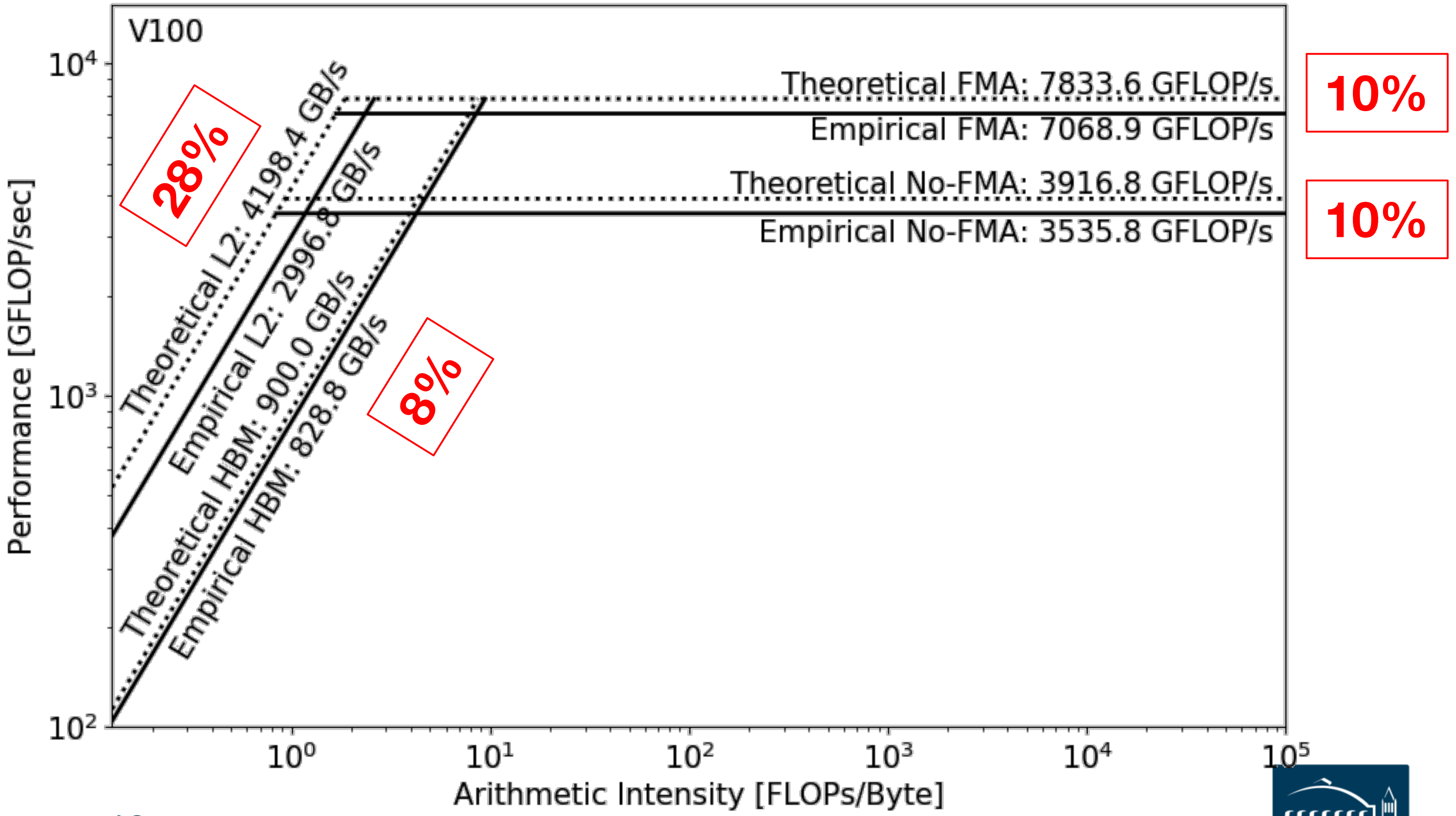
NVIDIA V100 -- Voltar at UOregon

Discrepancy Empirical vs. Theoretical

- Theoretical FP64 **compute** ceilings on V100:
  - FMA: 80 SMs x 32 FP64 cores x 1.53 GHz x 2 = 7.83 TFLOP/s
  - no FMA: 80 SMs x 32 FP64 cores x 1.53 GHz = 3.92 TFLOP/s
- Theoretical **memory** bandwidths on V100:
  - HBM: 900 GB/s
  - L2: ~4.1 TB/s
  - L1: ~14 TB/s

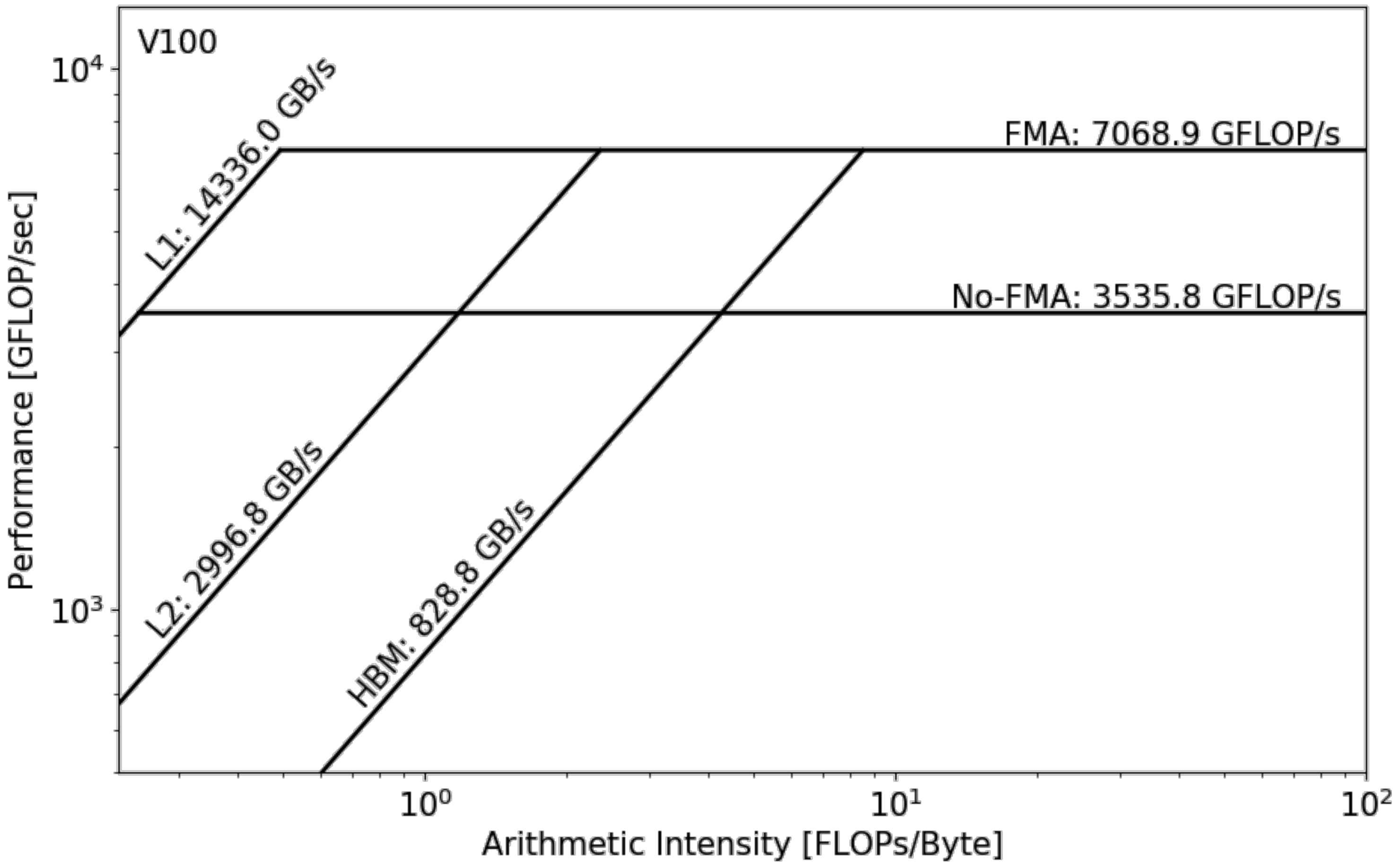
- **You may never achieve 7.8 TFLOP/s**

- **You may be closer to the ceiling than you think you are**

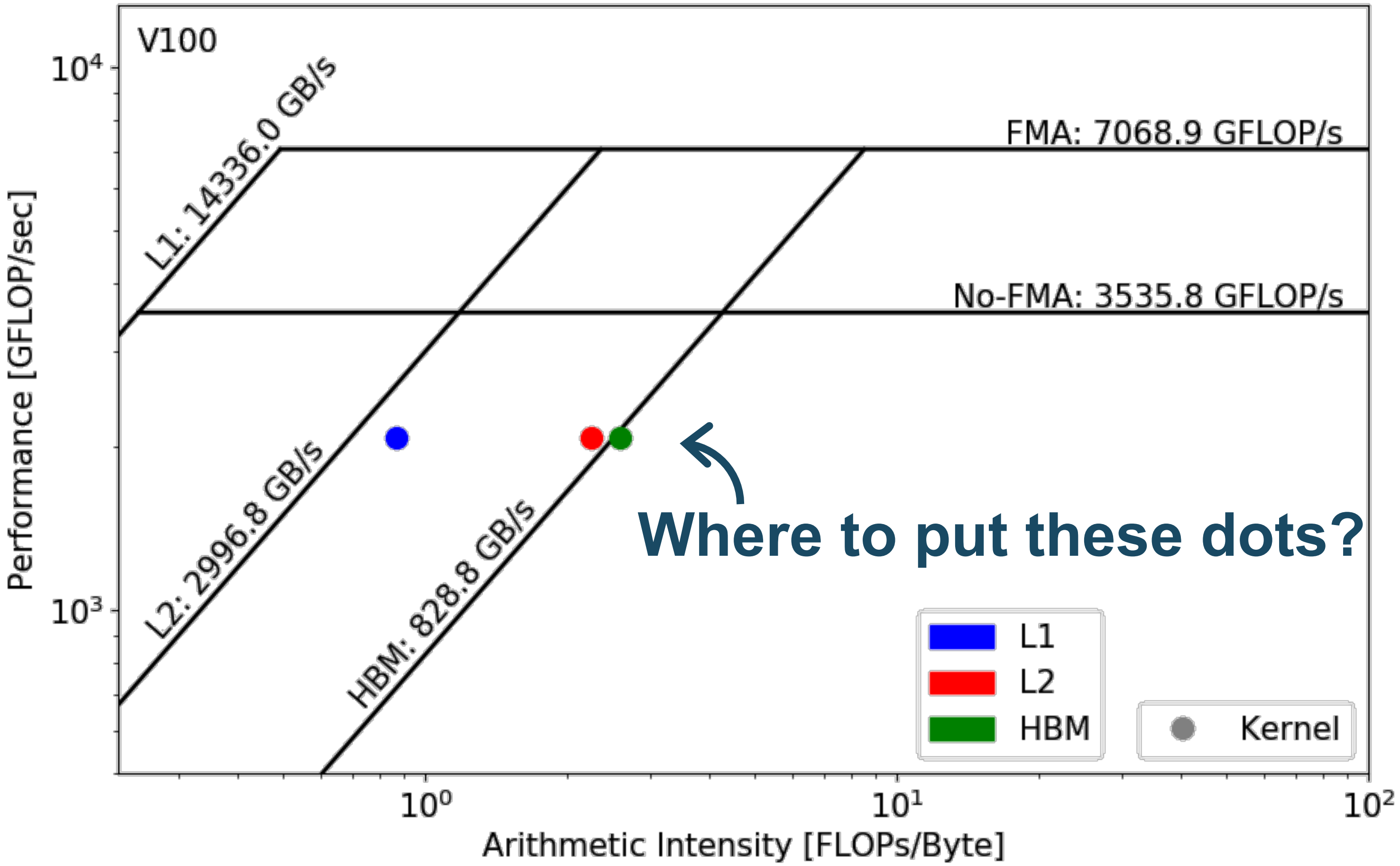


Voltar at UOregon

Step 2. Collect Application Performance

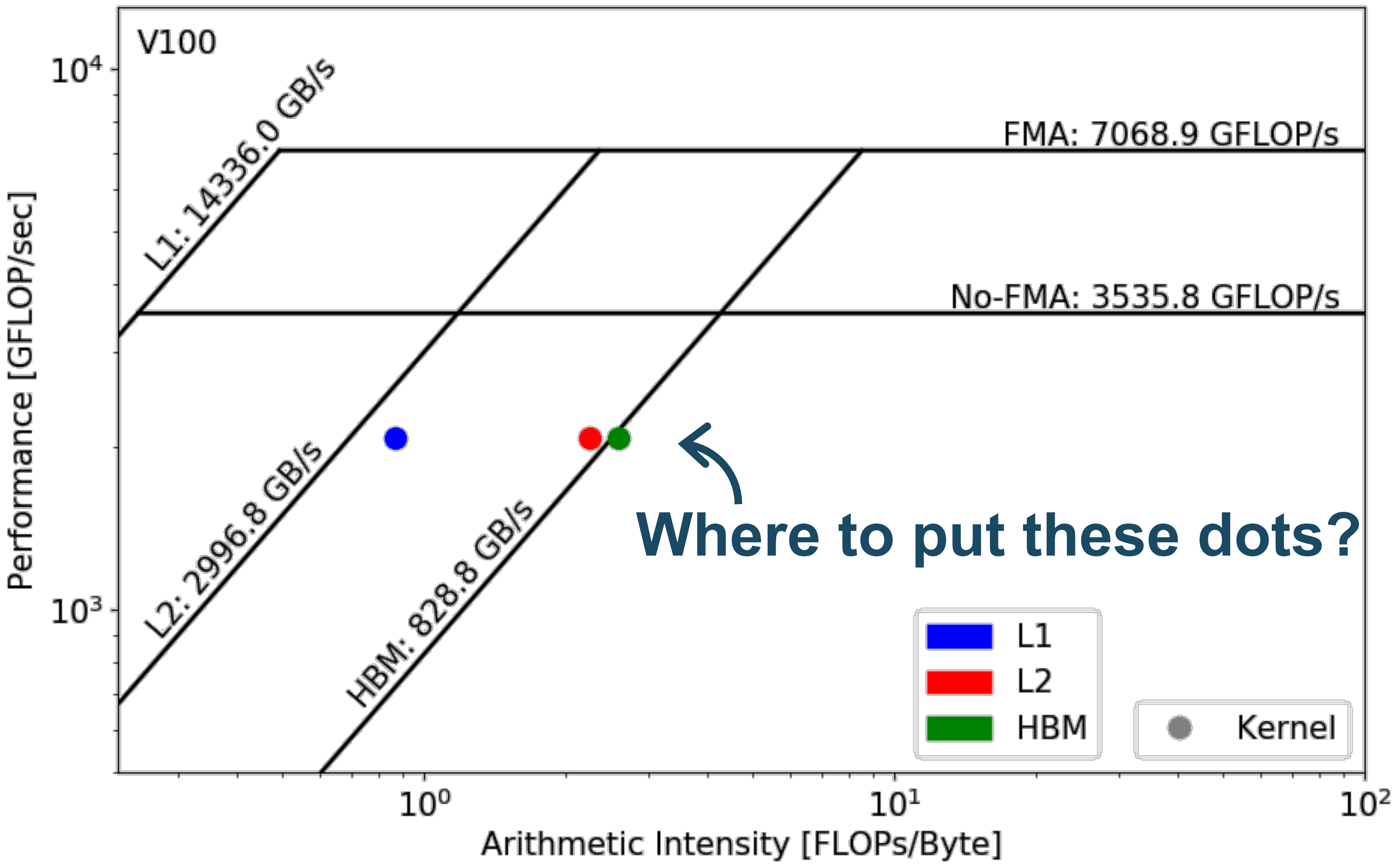


Step 2. Collect Application Performance



Where to put these dots?

Step 2. Collect Application Performance



Require three raw measurements:

- **Runtime**
- **FLOPs**
- **Bytes (on each cache level)**

to calculate AI and GFLOP/s:

$$\underset{\text{(FLOPs/Byte)}}{\text{Arithmetic Intensity}} = \frac{\textit{nvprof}\ \text{FLOPs}}{\textit{nvprof}\ \text{Data Movement}}$$

$$\underset{\text{(GFLOP/s)}}{\text{Performance}} = \frac{\textit{nvprof}\ \text{FLOPs}}{\text{Runtime}}$$

Collect Application Performance

- Runtime:
  - Time per invocation of a kernel

    `nvprof --print-gpu-trace ./application`
  - Average time over multiple invocations

    `nvprof --print-gpu-summary ./application`
  - Same kernel with different input parameters are grouped separately

- FLOPs:
  - Predication aware and complex-operation aware (such as divides)
  - `nvprof --kernels 'kernel_name' --metrics 'flop_count_xx' ./application`
  - e.g. `flop_count_{dp/dp_add/dp_mul/dp_fma, sp*, hp*}`

Collect Application Performance

- Bytes for different cache levels in order to construct hierarchical Roofline:
  - Bytes = (read transactions + write transactions) x transaction size
  - `nvprof --kernels 'kernel_name' --metrics 'metric_name' ./application`

Level	Metrics	Transaction Size
First Level Cache*	`gld_transactions`, `gst_transactions`, `atomic_transactions`, `local_load_transactions`, `local_store_transactions`, `shared_load_transactions`, `shared_store_transactions`	32B
Second Level Cache	`l2_read_transactions`, `l2_write_transactions`	32B
Device Memory	`dram_read_transactions`, `dram_write_transactions`	32B
System Memory	`system_read_transactions`, `system_write_transactions`	32B

- Note: surface and texture transactions are ignored here for simplicity (HPC applications)

Example Output

```
[cjyang@voltar source]$ nvprof --kernels "1:7:smooth_kernel:1" --metrics
flop_count_dp --metrics gld_transactions --metrics gst_transactions --
metrics l2_read_transactions --metrics l2_write_transactions --metrics
dram_read_transactions --metrics dram_write_transactions --metrics
sysmem_read_bytes --metrics sysmem_write_bytes ./hpgmg-fv-fp 5 8
```

context : stream : kernel : invocation

- Export to CSV: `--csv -o nvprof.out`

```
Invocations                               Metric Name                             Metric Description         Min         Max         Avg
Device "Tesla V100-PCIE-16GB (0)"
    Kernel: void smooth_kernel<int=6, int=32, int=4, int=8>(level_type, int, int, double, double, int, double*, double*)
          1                             flop_count_dp   Floating Point Operations(Double Precision)    30277632    30277632    30277632
          1                          gld_transactions                      Global Load Transactions     4280320     4280320     4280320
          1                          gst_transactions                     Global Store Transactions       73728       73728       73728
          1                      l2_read_transactions                          L2 Read Transactions      890596      890596      890596
          1                     l2_write_transactions                         L2 Write Transactions       85927       85927       85927
          1                    dram_read_transactions               Device Memory Read Transactions      702911      702911      702911
          1                   dram_write_transactions              Device Memory Write Transactions      151487      151487      151487
          1                         sysmem_read_bytes                      System Memory Read Bytes           0           0           0
          1                        sysmem_write_bytes                     System Memory Write Bytes         160         160         160
```

Step 3. Plot Roofline with Python

- Calculate Arithmetic Intensity and GFLOP/s performance
  - x coordinate: Arithmetic Intensity
  - y coordinate: GFLOP/s performance

$$\underset{\text{(GFLOP/s)}}{\text{Performance}} = \frac{nvprof\ \text{FLOPs}}{\text{Runtime}}, \quad \underset{\text{(FLOPs/Byte)}}{\text{Arithmetic Intensity}} = \frac{nvprof\ \text{FLOPs}}{nvprof\ \text{Data Movement}}$$

- Plot Roofline with Python Matplotlib
  - Example scripts:
  - https://github.com/cyanguwa/nersc-roofline/tree/master/Plotting
  - Tweak as needed for more complex Rooflines

Plot Roofline with Python

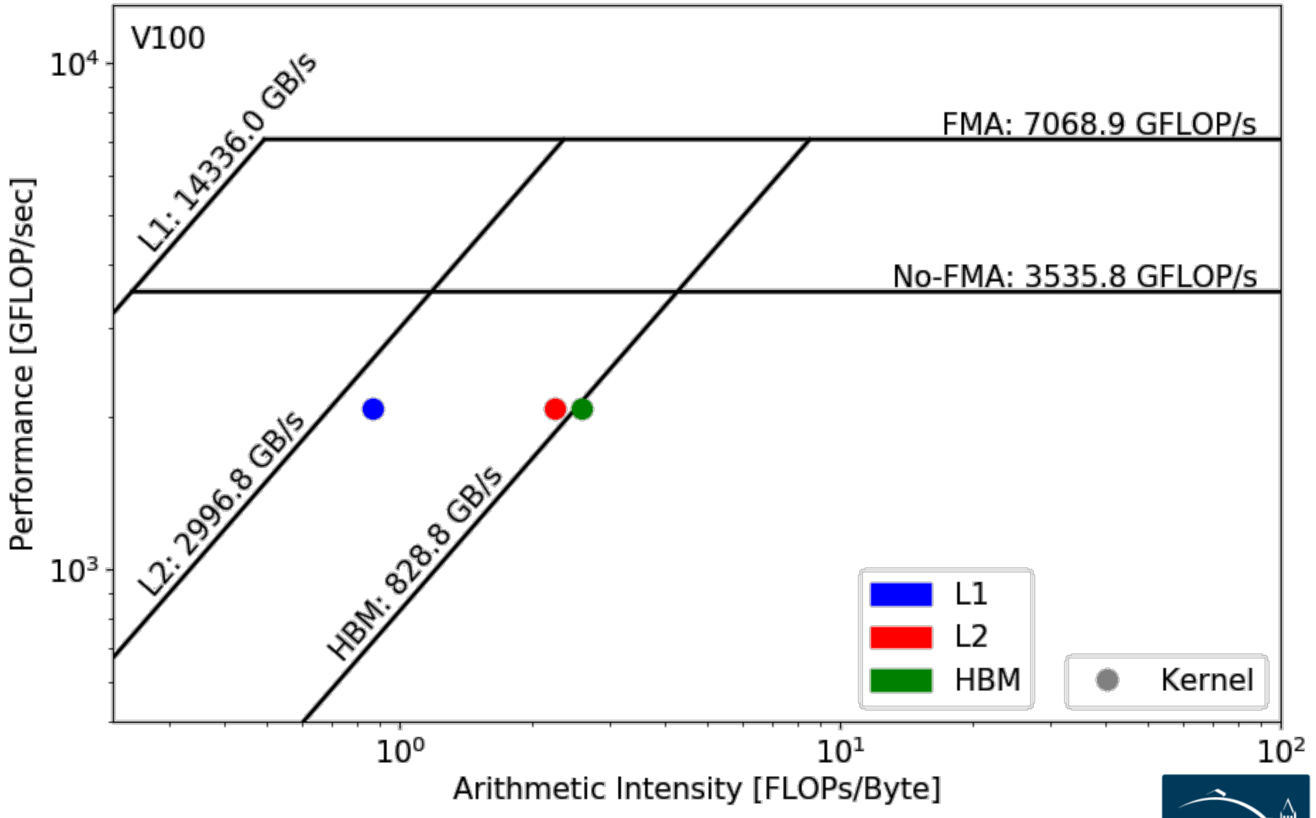
- Quick example: `plot_roofline.py data.txt`

- Accepts space-delimited list for values
- Use quotes to separate names/labels

```
data.txt

# all data is space delimited
memroofs 14336.0 2996.8 828.758
mem_roof_names 'L1' 'L2' 'HBM'
comproofs 7068.86 3535.79
comp_roof_names 'FMA' 'No-FMA'

# omit the following if only plotting roofs
# AI: arithmetic intensity; GFLOPs: performance
AI 0.87 2.25 2.58
GFLOPs 2085.756683
labels 'Kernel'
```



Recap: Methodology to Construct Roofline

1. **Collect Roofline ceilings**
   - ERT: https://bitbucket.org/berkeleylab/cs-roofline-toolkit
   - compute (FMA/no FMA) and bandwidth (DRAM, L2, …)
2. **Collect application performance**
   - nvprof: `--metrics`, `--events`, `--print-gpu-trace`
   - FLOPs, bytes (DRAM, L2, …), runtime
3. **Plot Roofline with Python Matplotlib**
   - arithmetic intensity, GFLOP/s performance, ceilings
   - example scripts: https://github.com/cyanguwa/nersc-roofline

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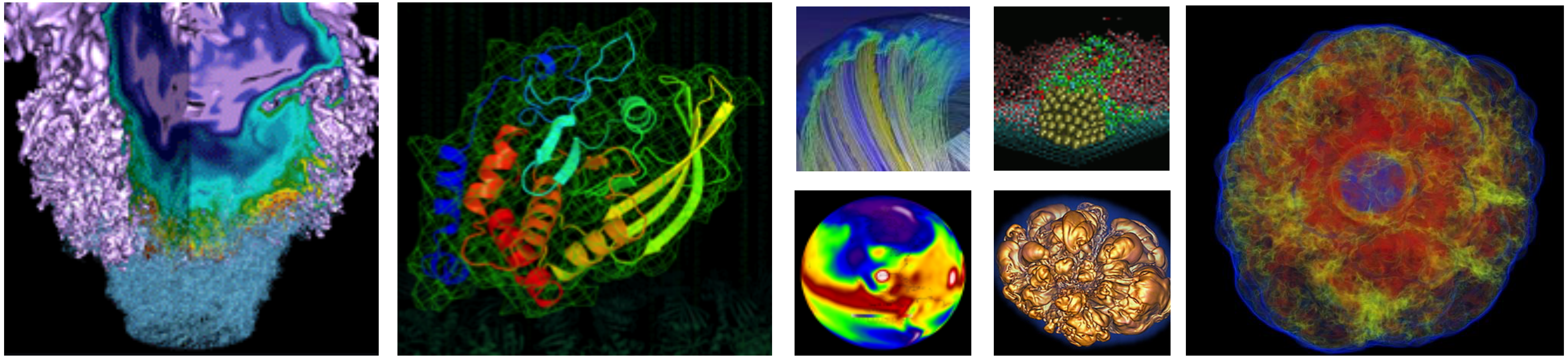

Roofline Analysis with Use Cases

Code Example 1: GPP

- GPP (General Plasmon Pole) kernel from BerkeleyGW (Material Science)
- https://github.com/cyanguwa/BerkeleyGW-GPP
- Medium problem size: 512 2 32768 20

- Tensor-contraction, abundant parallelism, large reductions
- Low FMA counts, divides, complex double data type, HBM data 1.5GB

Pseudo Code

```
do band = 1, nbands              #blockIdx.x
   do igp = 1, ngpown            #blockIdx.y
      do ig = 1, ncouls          #threadIdx.x
         do iw = 1, nw           #unrolled
            compute; reductions
```

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Code Example 1: GPP

- Three experiments:

Vary `nw` from 1 to 6	To study impact of **varying Arithmetic Intensity** on performance
Compile w/wo FMA	To study impact of **instruction mix** on performance on performance
Stride `ig` loop	To study impact of **suboptimal memory coalescing** on performance

- Note that `nvprof` has already taken care of
  - Appropriate counting of FLOPs for complex instructions
    - div, exp, log and sin/cos should be counted as multiple FLOPs rather than 1
  - Appropriate counting of FLOPs for predicated-out threads
    - FLOPs are only counted on non-predicated threads

Code Example 1: GPP

- Highly parameterizable
    1. Varying **`nw`** from 1 to 6 to increase arithmetic intensity
        - FLOPs increases, but data movement stays (at least for HBM)

Pseudo Code

```
do band = 1, nbands                  #blockIdx.x
   do igp = 1, ngpown                #blockIdx.y
      do ig = 1, ncouls              #threadsIdx.x
         do iw = 1, nw               #unrolled
            compute; reductions
```

2. Compiling with and without FMA
    - `-fmad=true/false`

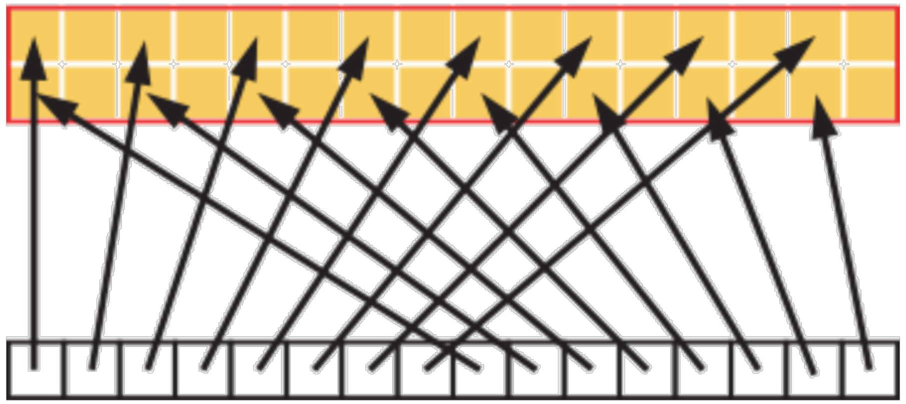
Code Example 1: GPP

- Highly parameterizable
    3. Striding **`ig`** loop to analyze impact of suboptimal memory coalescing
        - Split **`ig`** loop to two loops and place the ‘blocking’ loop outside

Pseudo Code

```
do band = 1, nbands                    #blockIdx.x
   do igp = 1, ngpown                  #blockIdx.y
      do igs = 0, stride - 1
         do ig = 1, ncouls/stride #threadIdx.x
            do iw = 1, nw          #unrolled
               compute; reductions
```

Stride 2

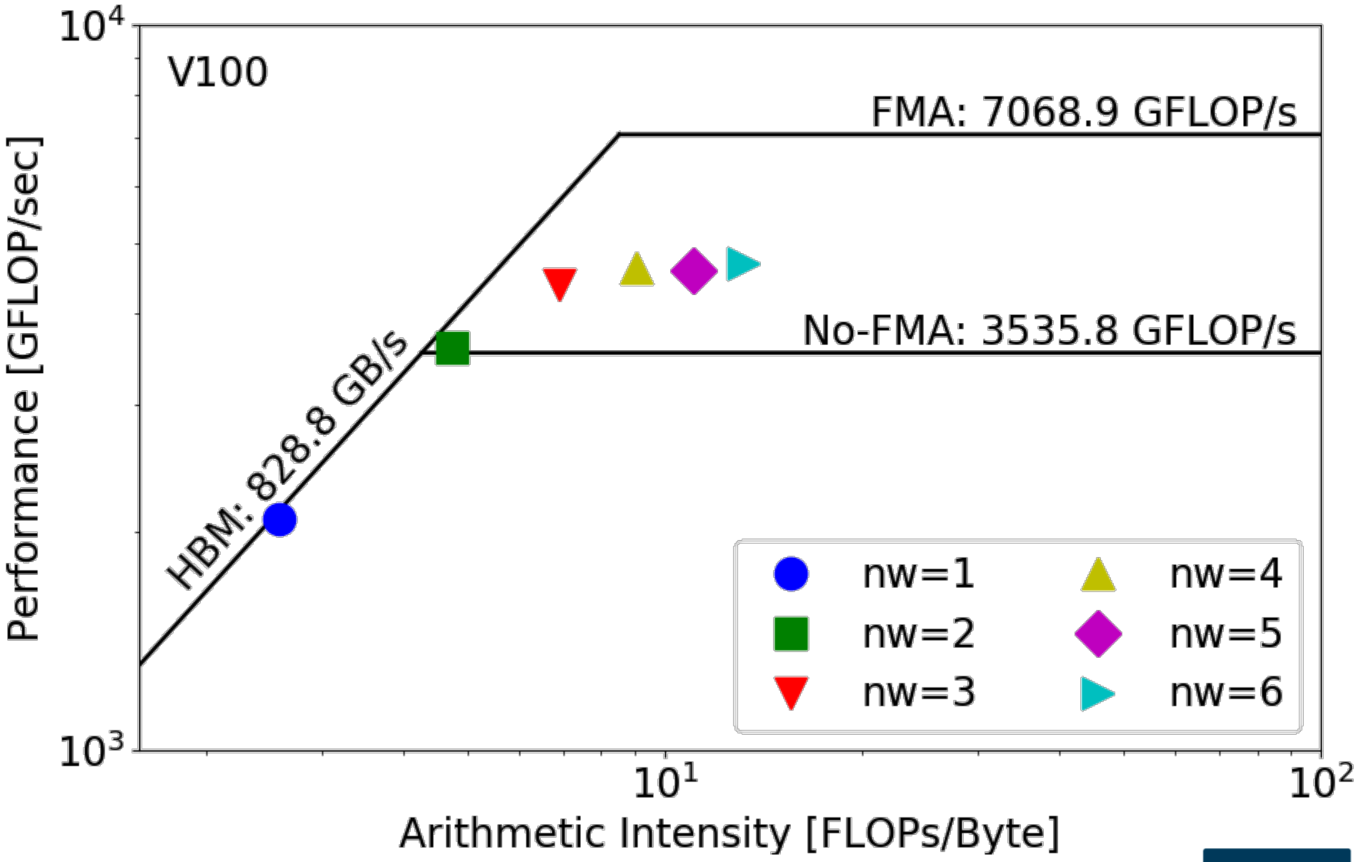


Code Example 1: GPP

- **Experiments 1:** study the impact of varying AI on performance

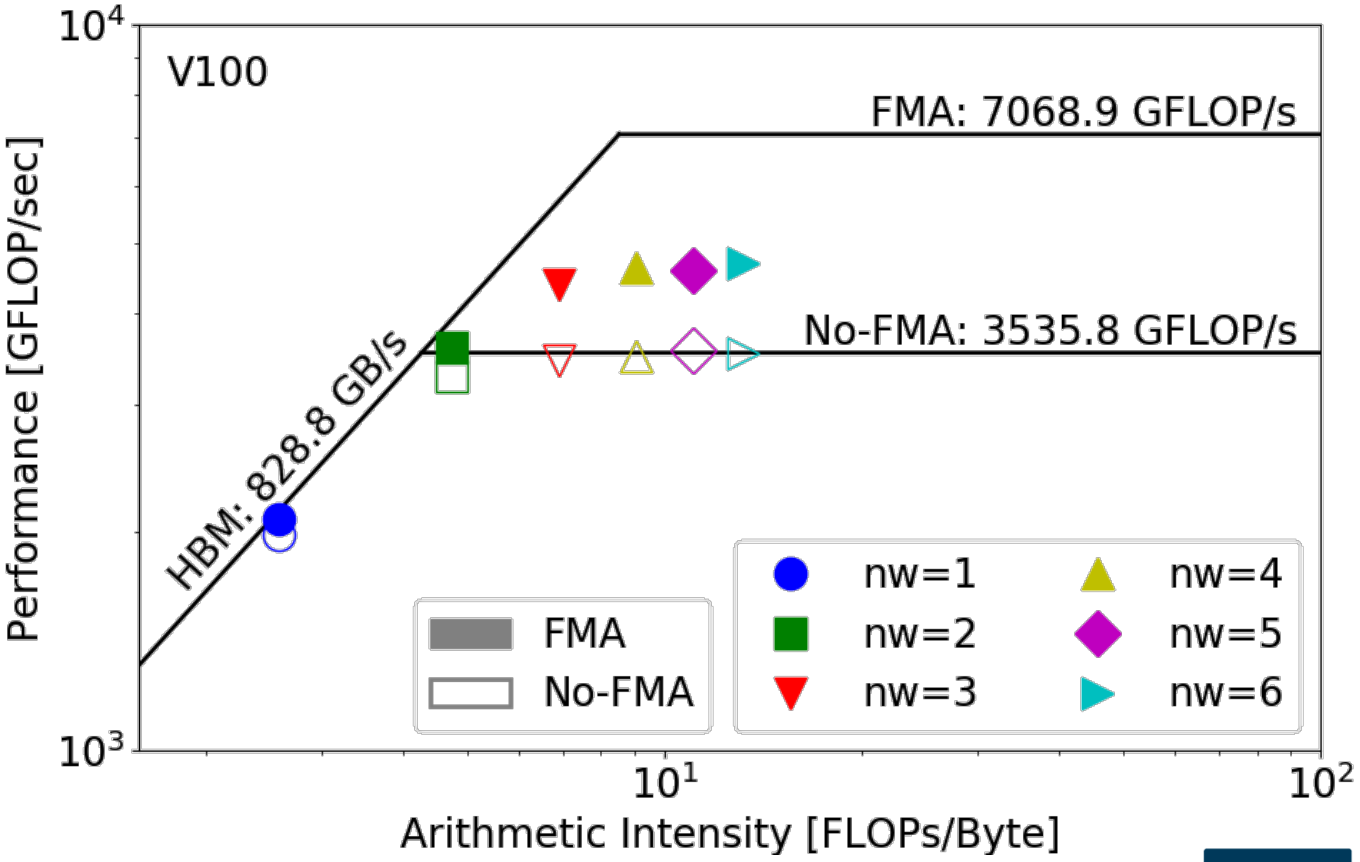
- HBM Roofline, i.e. bytes are HBM bytes
  - AI increases as **`nw`** grows
  - GPP moves from a bandwidth bound region to a compute bound region

- Roofline captures the change in AI



Code Example 1: GPP

- **Experiments 1 & 2:** study the impact of instruction mix on performance
- HBM Roofline, i.e. bytes are HBM bytes
  - No-FMA performance converges to the no-FMA ceiling, but FMA performance is still far from the FMA ceiling
  - Not reaching FMA ceiling due to lack of FMA instructions
- Roofline captures effects of instruction mix

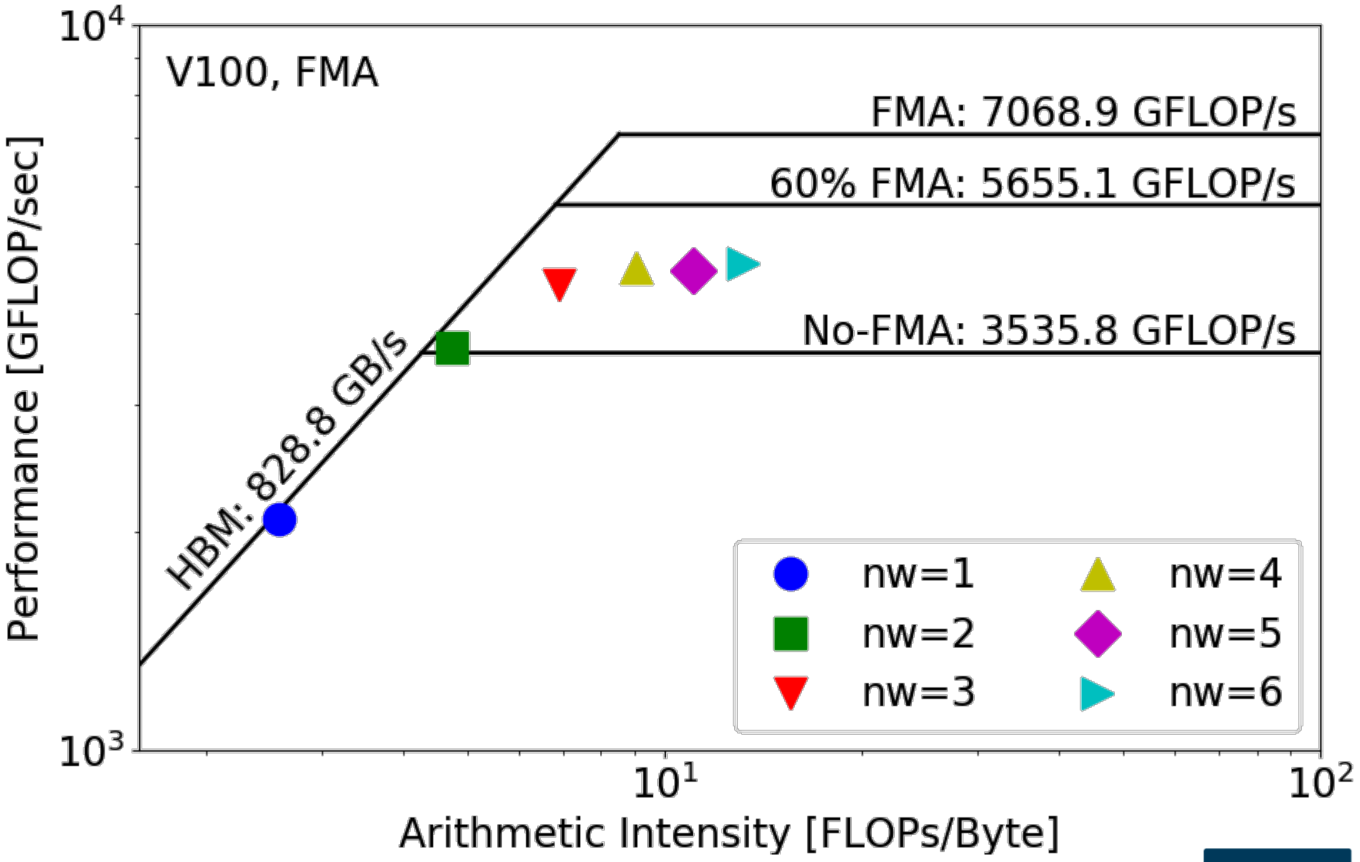


Code Example 1: GPP

- **Experiments 1 & 2:** study the impact of instruction mix on performance
- At **nw**=6, GPP has $\alpha = \frac{\text{FMA FP64 instr.}}{\text{FMA FP64 instr.} + \text{non-FMA FP64 instr.}} = 60\%$ of FMA instructions
- Expected performance is

  $\beta = \frac{\alpha \times 2 + (1-\alpha)}{2} = 80\%$ of compute peak.

  But at **nw**=6, GPP is only achieving **66%**.
- Other FP/non-FP instructions may be taking up the instruction issue/execution pipeline
- Partial Roofline can show you the headroom

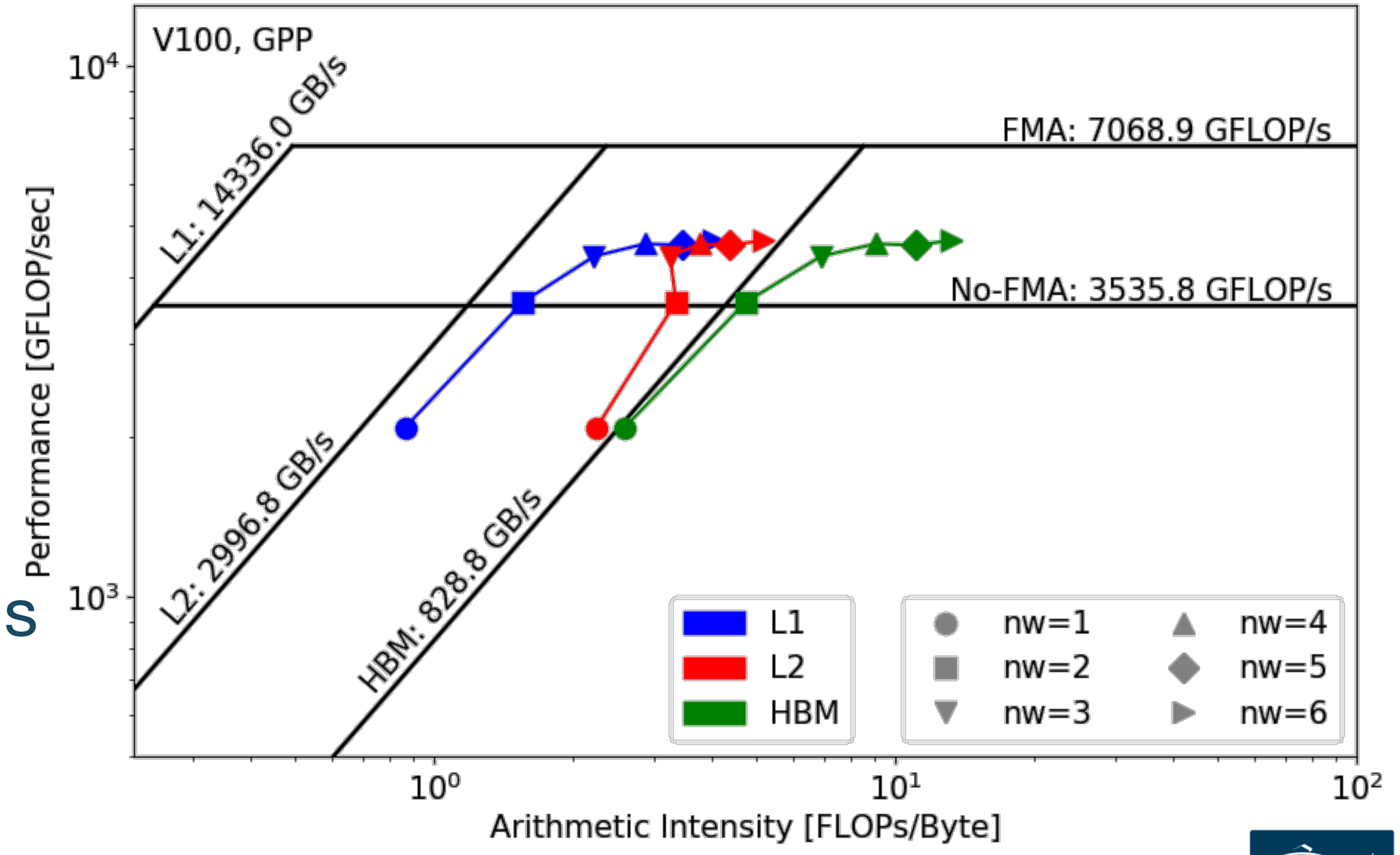


Code Example 1: GPP

- **Experiments 1 & 2:** What else is going on?

- Hierarchical Roofline, i.e. bytes are HBM, L2 and unified L1 cache bytes
  - GPP is HBM bound at low **`nw`**'s and compute bound at high **`nw`**'s
  - FLOPs ∝ **`nw`**
  - HBM bytes: constant
  - L2 bytes: increasing at $\alpha > 1$
  - L1 bytes: constant
  - Spike in L2 curve at **`nw`**=2, 3

- Hierarchical Roofline captures more details about cache locality

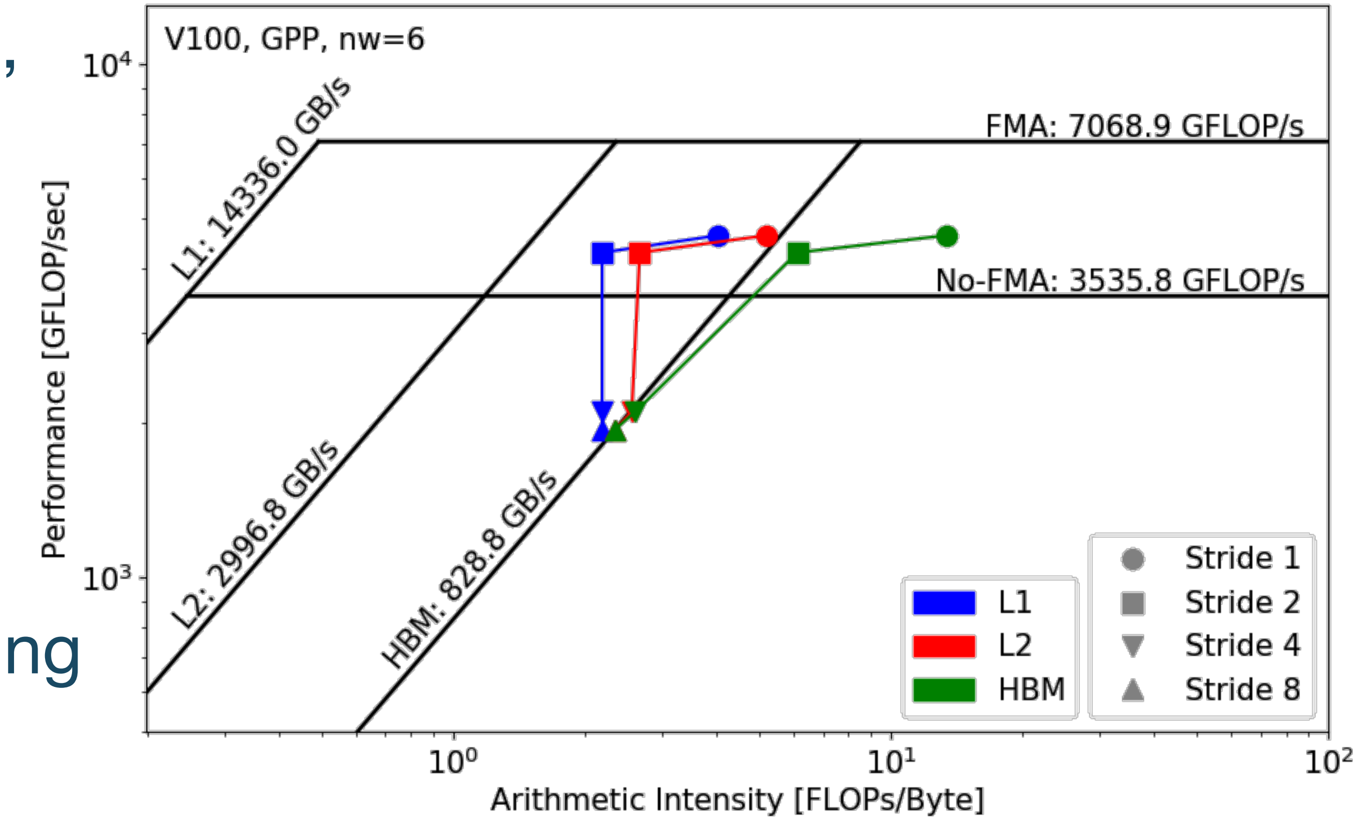


Code Example 1: GPP

- **Experiment 3:** study the effects of suboptimal memory coalescing
  - `nw`=6

- Hierarchical Roofline, i.e. bytes are HBM, L2 and unified L1 cache bytes
  - L1/L2 bytes doubles from stride 1 to 2, but stays almost constant afterwards
  - at `nw`=6, GPP moves from compute bound to bandwidth bound
  - Eventually all dots converge to HBM

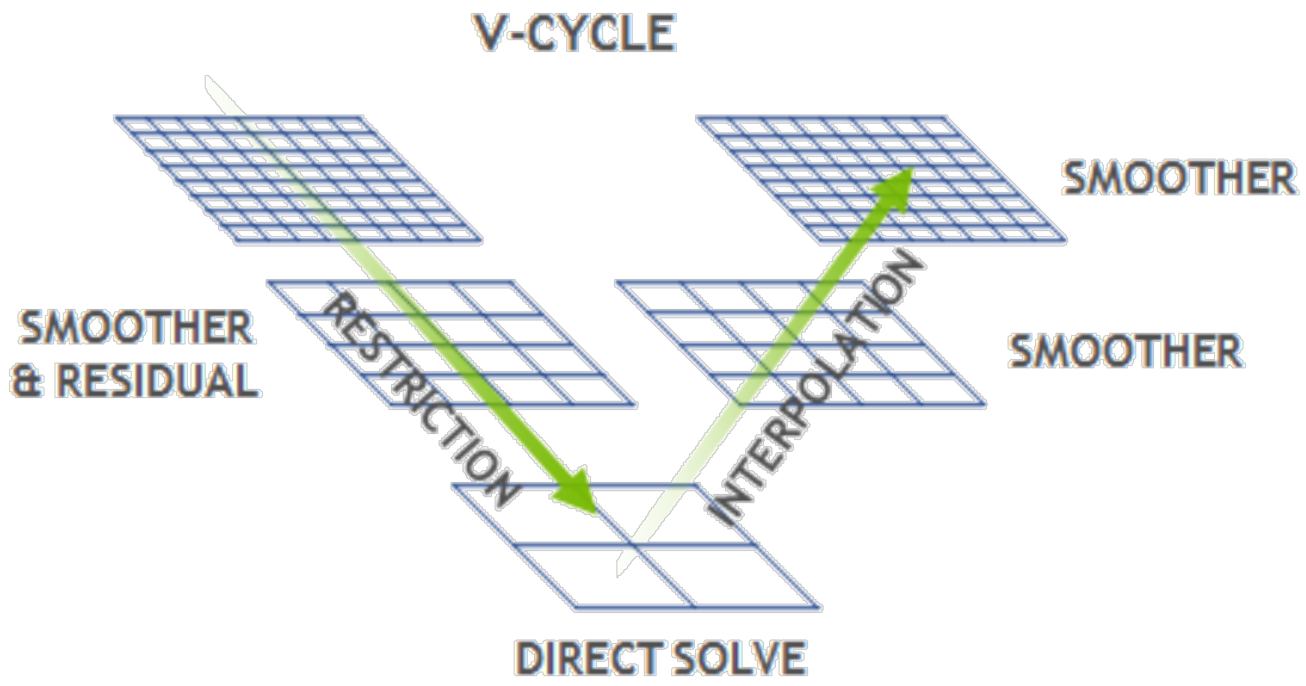
- Roofline captures effects of memory coalescing

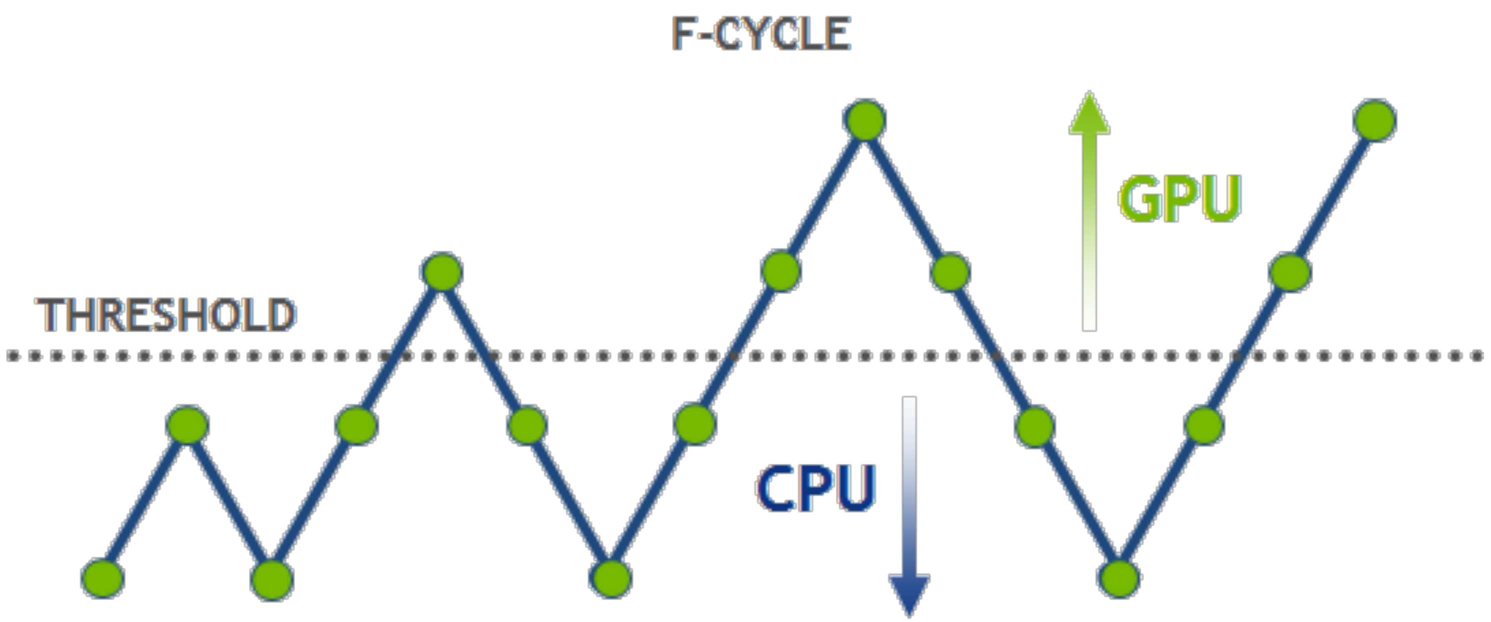


Code Example 2: HPGMG

- HPGMG (High-performance Geometric Multigrid) from Adaptive Mesh Refinement codes
- https://bitbucket.org/nsakharnykh/hpgmg-cuda

- Stencil code, F-cycles and V-cycles, GSRB smoother kernel (Gauss-Seidel Red-Black)



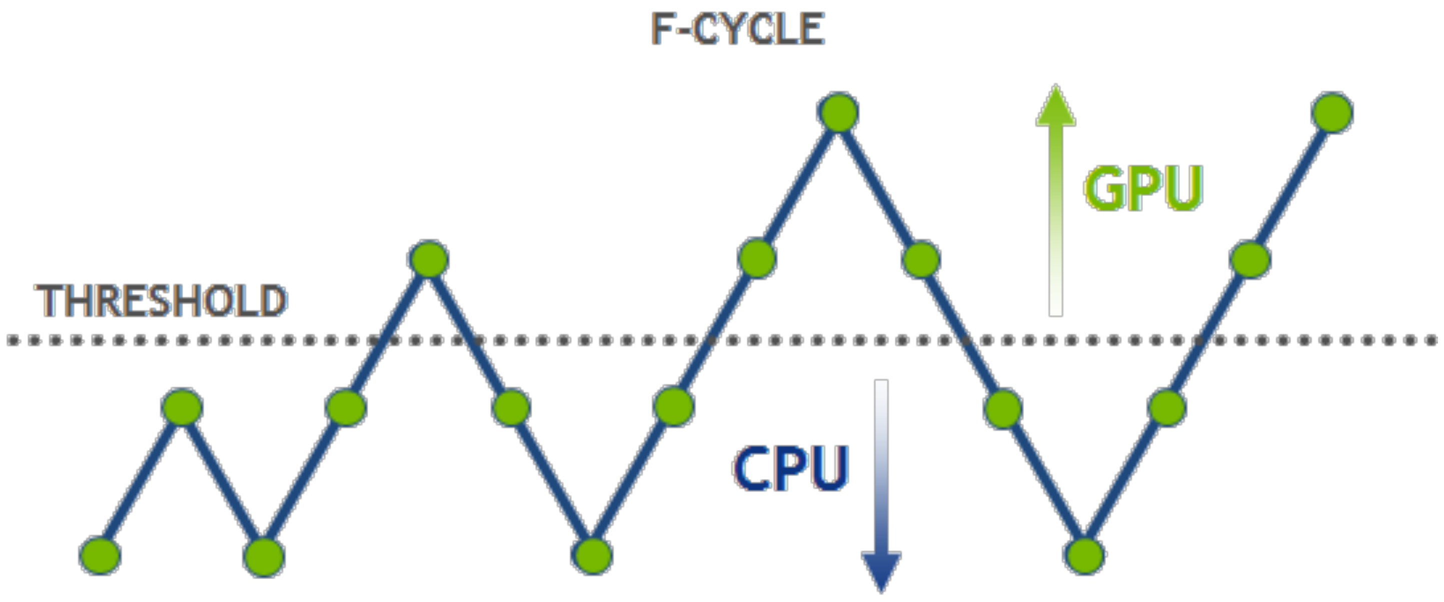


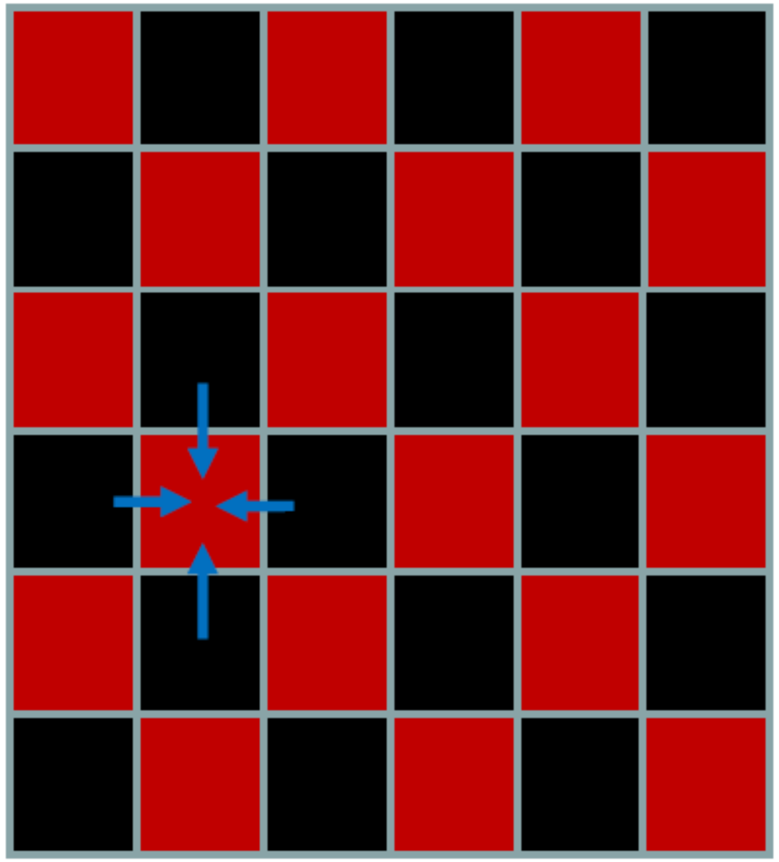
HPGMG. https://devblogs.nvidia.com/high-performance-geometric-multi-grid-gpu-acceleration/

Code Example 2: HPGMG

- Hybrid GPU and CPU code
  - Example: `hpgmg-fv 7 8`
  - 128^3 box x 8, Level 5-8 run on GPU, Level 1-4 on CPU

- Three versions of GSRB kernel
  - GSRB_FP, GSRB_BRANCH, GSRB_STRIDE2

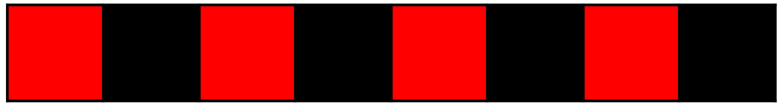




Code Example 2: HPGMG

```
GSRB_FP

for(int k=klo; k<(klo+kdim); k++){
  const int ijk = i + j*jStride + k*kStride;
  const double *__restrict__ RedBlack =
      level.RedBlack_FP + ghosts*(1+jStride)
      +((k^color000)&1)*kStride;
  const double Ax = apply_op_ijk();
  const double lambda = Dinv_ijk();
  const int ij = i + j*jStride;
  xo[ijk] = X(ijk) + RedBlack[ij]*lambda*(rhs[ijk]-Ax);
}
```



8 elements

Sweep

1	0	1	0	1	0	1	0

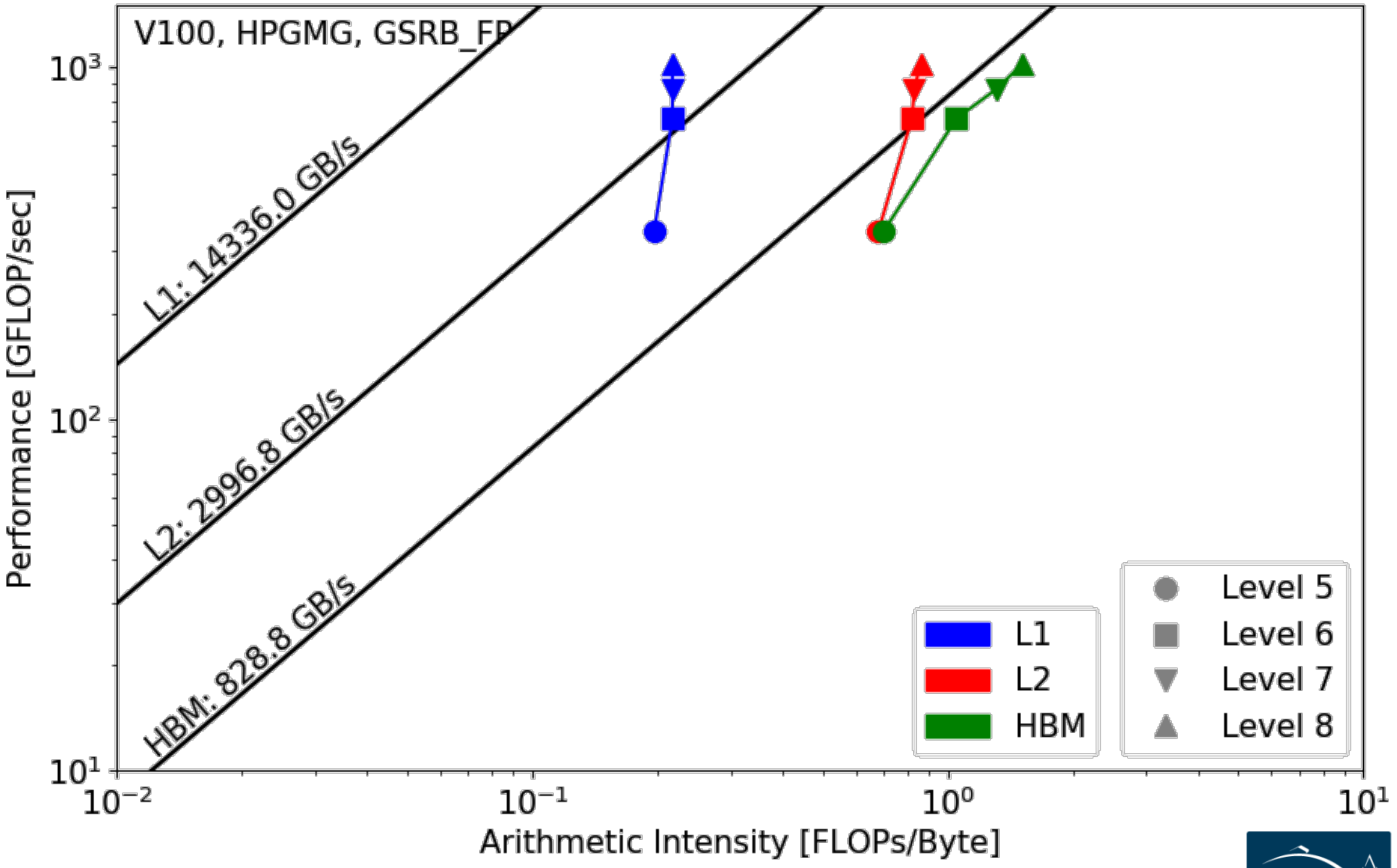
8 threads

Code Example 2: HPGMG

GSRB_FP

- Hierarchical Roofline, i.e. bytes are HBM, L2 and unified L1 cache bytes

- Highly bandwidth bound, inherent to stencil codes
- From Level 5 to Level 8:
  - AI slightly increases due to better Surface: Volume ratio
  - More HBM bound as more data is read in



- Roofline captures computational characteristics of the algorithm

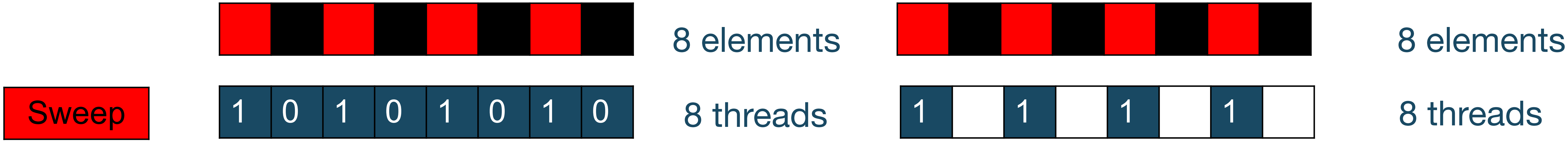
Code Example 2: HPGMG

GSRB_FP

```
for(int k=klo; k<(klo+kdim); k++){
  const int ijk = i + j*jStride + k*kStride;
  const double *__restrict__ RedBlack =
      level.RedBlack_FP + ghosts*(1+jStride)
      +((k^color000)&1)*kStride;
  const double Ax = apply_op_ijk();
  const double lambda = Dinv_ijk();
  const int ij = i + j*jStride;
  xo[ijk] = X(ijk) + RedBlack[ij]*lambda*(rhs[ijk]-Ax);
}
```

GSRB_BRANCH

```
for(int k=klo; k<klo+kdim; k++){
  const int ijk = i + j*jStride + k*kStride;
  if(((i^j^k^color000^1)&1)){
    const double Ax = apply_op_ijk();
    const double lambda = Dinv_ijk();
    xo[ijk] = X(ijk) + lambda*(rhs[ijk]-Ax);
  }else{
    xo[ijk] = X(ijk);
  }
}
```

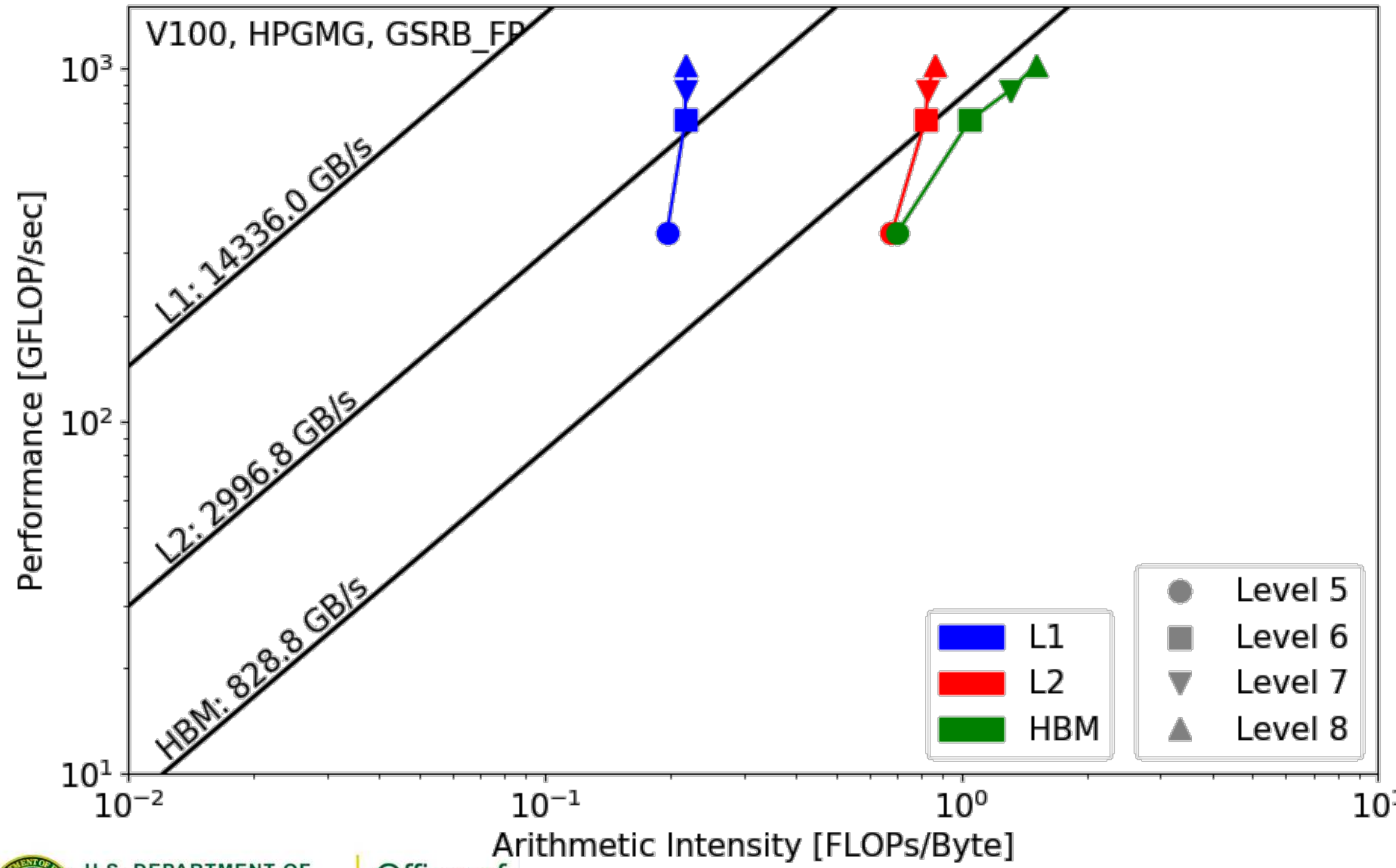


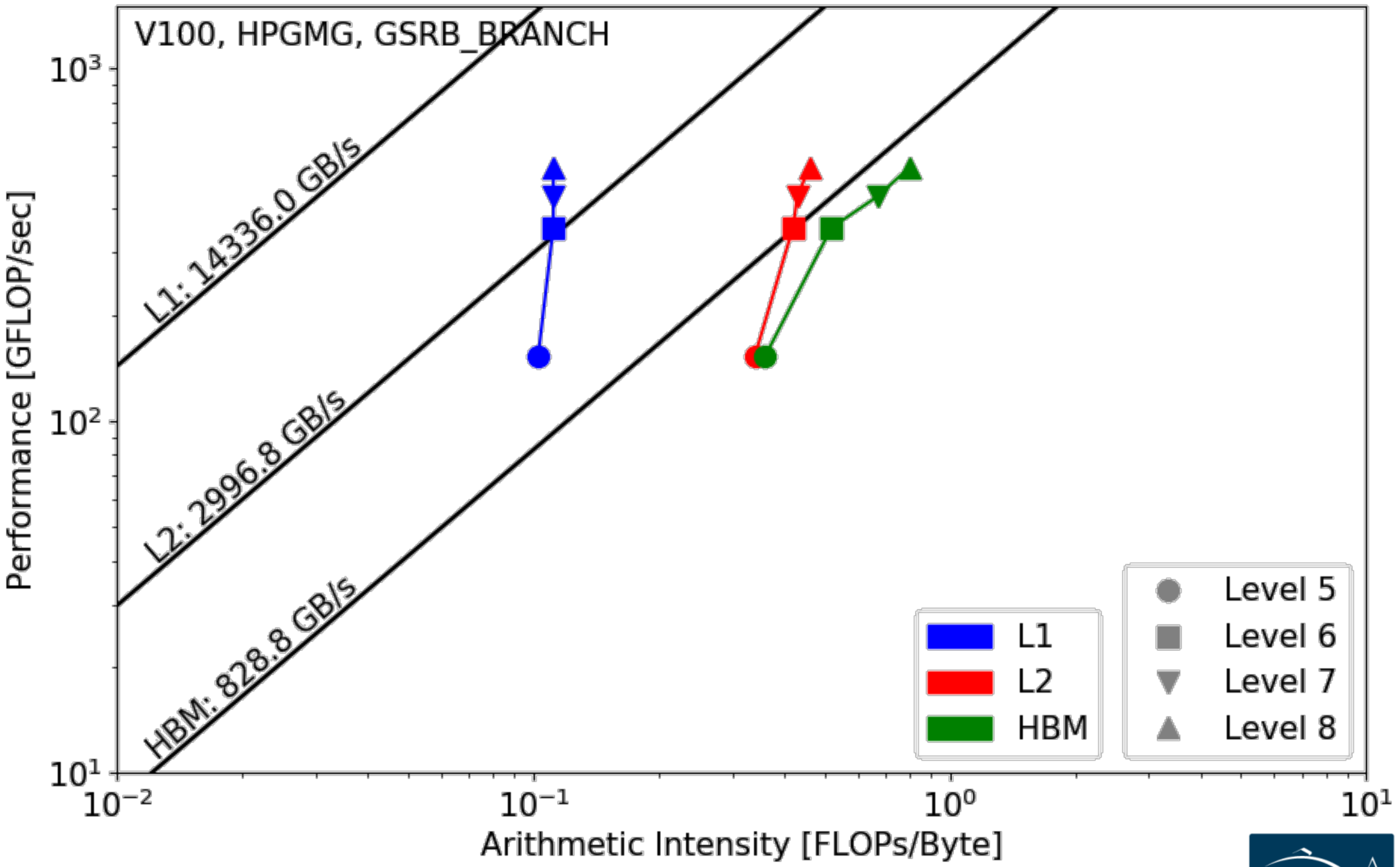
- GSRB_BRANCH has half the FLOPs as GSRB_FP but the same HBM/L1/L2 bytes

Code Example 2: HPGMG

GSRB_FP vs. GSRB_BRANCH

- FLOPs halves, bytes doesn't change, thus AI halves and GFLOP/s halves
- Runtime is comparable even though GFLOP/s has halved
- Same number of threads occupied, only with half predicated in GSRB_BRANCH

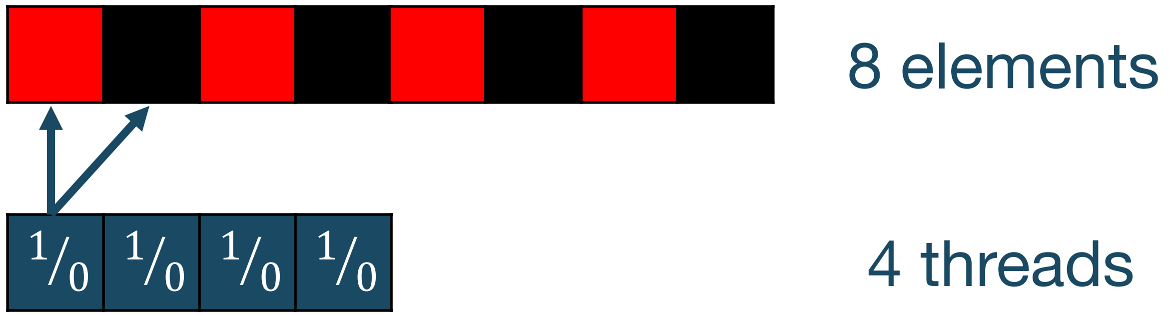




Code Example 2: HPGMG

```
GSRB_STRIDE2

for(int k=klo; k<klo+kdim; k++){
  i = ilo +!((ilo^j^k^color000)&1) + threadIdx.x*2;
  if(i < ilo+idim){
    const int ijk = i + j*jStride + k*kStride;
    xo[ijk] = X(ijk);
  }
  i = ilo + ((ilo^j^k^color000)&1) + threadIdx.x*2;
  if(i < ilo+idim){
    const int ijk = i + j*jStride + k*kStride;
    const double Ax = apply_op_ijk();
    const double lambda = Dinv_ijk();
    xo[ijk] = X(ijk) + lambda*(rhs[ijk]-Ax);
  }
}
```

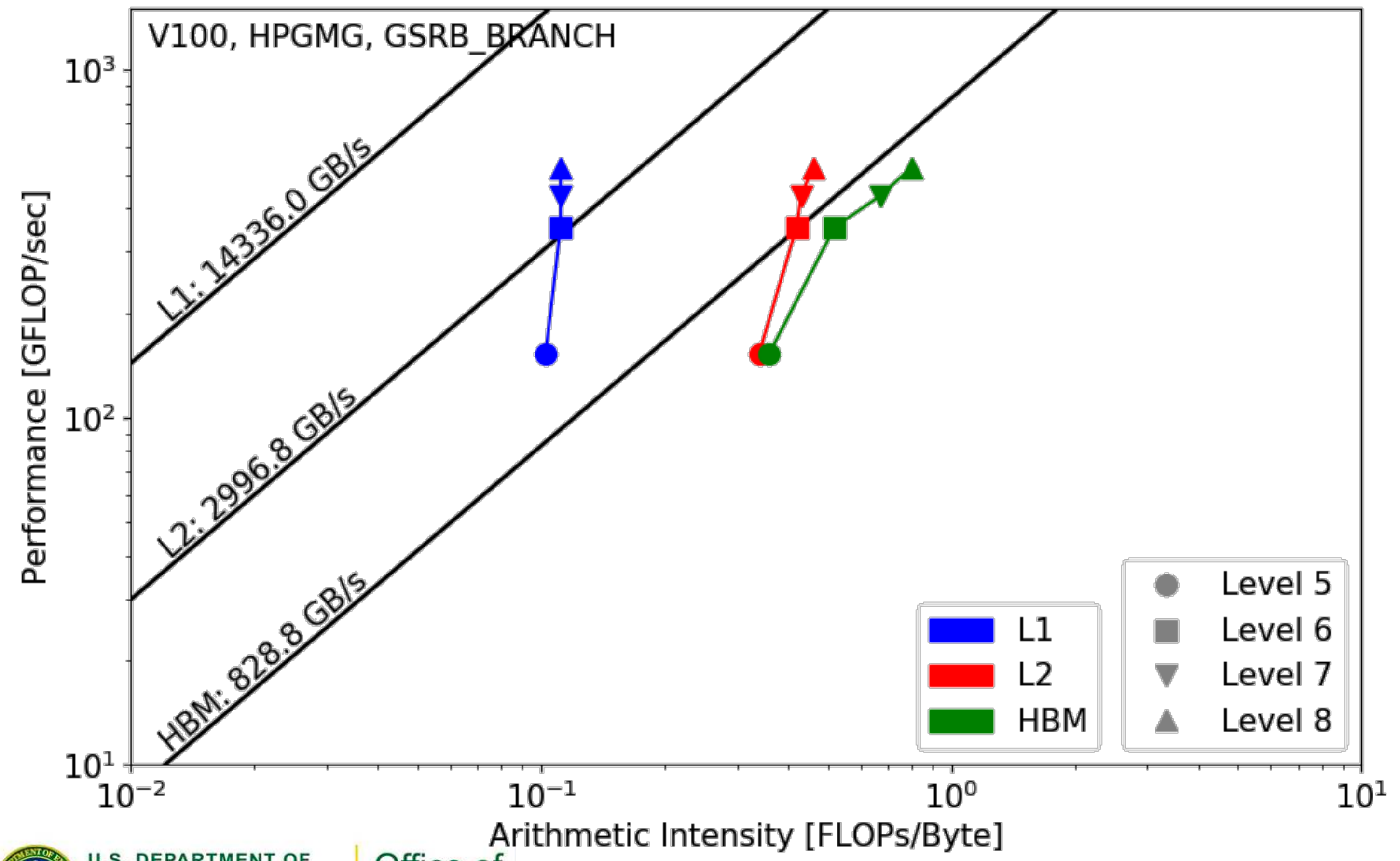


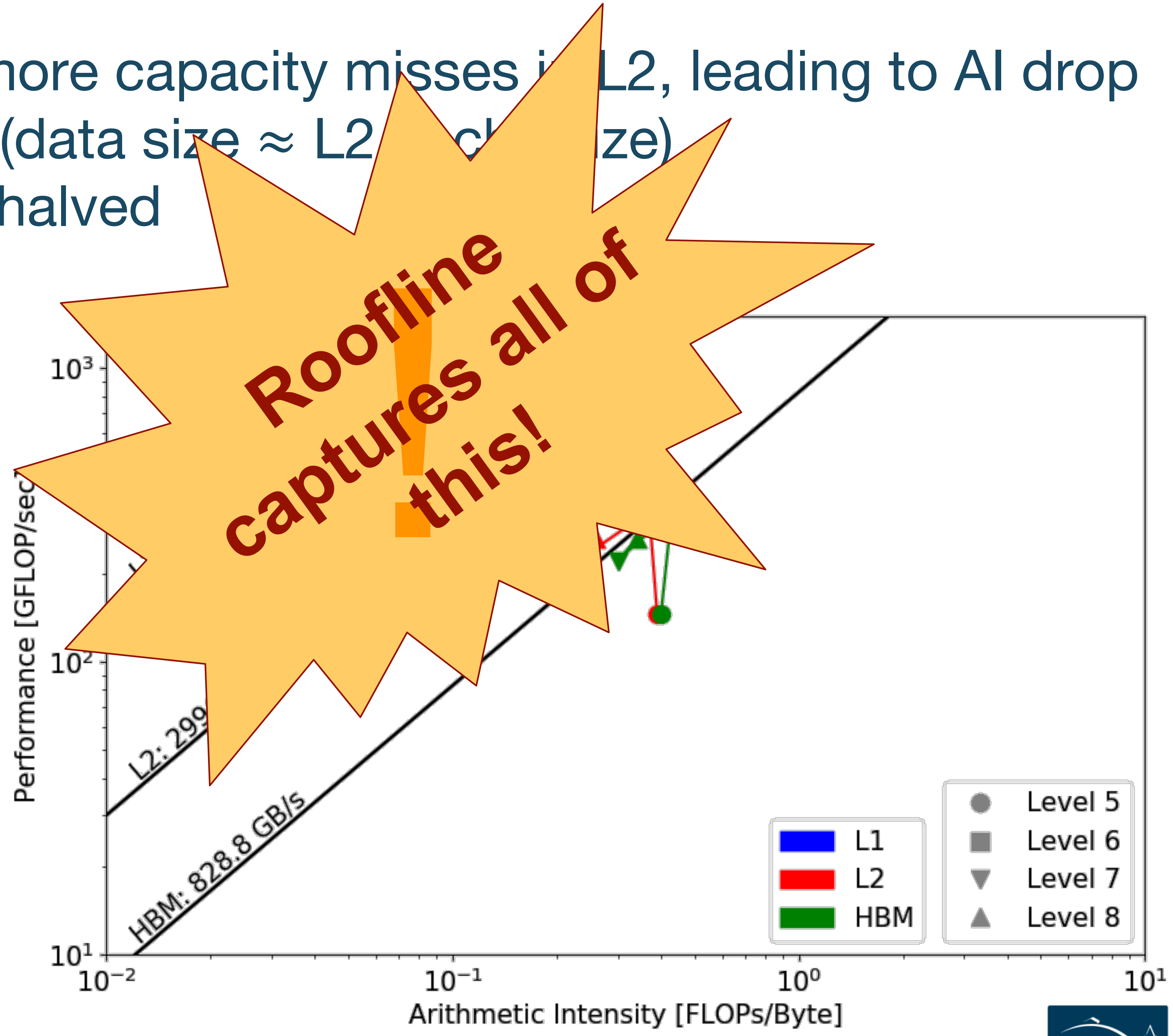
- GSRB_STRIDE2 should have the same FLOPs as GSRB_BRANCH, but same bytes? More writes than GSRB_BRANCH?

Code Example 2: HPGMG

GSRB_BRANCH vs. GSRB_STRIDE2

- Extra writes in GSRB_STRIDE2 cause more capacity misses in L2, leading to AI drop on L2 and DRAM, starting from Level 7 (data size ≈ L2 cache size)
- Runtime almost doubled and GFLOP/s halved





Conclusions

- Roofline can gracefully capture various aspects of application performance and architecture characteristics such as arithmetic intensity, instruction mix, memory coalescing and thread predication.

- The proposed methodology is effective in collecting machine characteristics and application data on NVIDIA GPUs to construct **hierarchical Roofline**.

- The Roofline model provides **insights** that profil
  - identify the most immediate bottleneck
  - prioritize optimization efforts
  - tell you when you can stop

A systematic and intuitive way of code optimization!

Reference

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Thank You!