

Tessellating NURBS with CUDA

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NURBS

- Non-Uniform Rational B-Splines
- Curved surfaces commonly used in DCC / CAD modeling
- Points on surface defined by basis functions, CVs, weights

$$S(u, v) = \frac{\sum_{i=0}^p \sum_{j=0}^q B_{i,m}(u) \cdot B_{j,n}(v) \cdot P_{i,j} \cdot w_{i,j}}{\sum_{i=0}^p \sum_{j=0}^q B_{i,m}(u) \cdot B_{j,n}(v) \cdot w_{i,j}}$$

- NURBS Basis $B_{i,k}$ defined recursively in terms of knot vector t_i

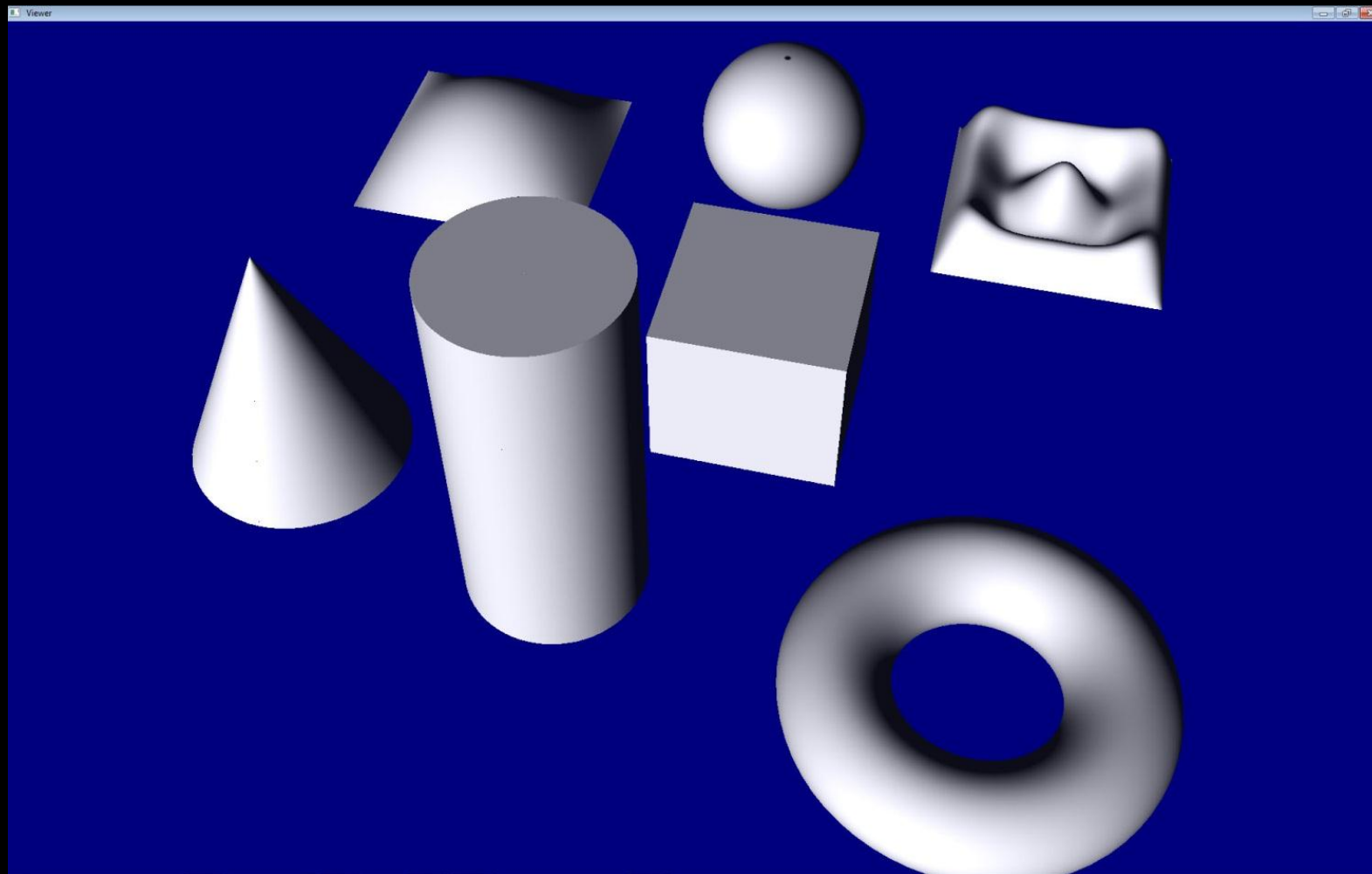
$$B_{i,0}(t) = \begin{cases} 1 & \text{if } t_i \leq t < t_{i+1} \\ 0 & \text{else} \end{cases}$$

$$\forall k > 0, B_{i,k}(t) = \frac{t - t_i}{t_{i+k} - t_i} B_{i,k-1}(t) + \frac{t_{i+k+1} - t}{t_{i+k+1} - t_{i+1}} B_{i+1,k-1}(t)$$

Tessellating NURBS with CUDA

- Efficient direct tessellation of NURBS surfaces
- Arbitrary order per surface
- Arbitrary knot vectors & number of patches
- Programmable UV tessellation patterns
- Programmable triangulation
- Enable trimmed surfaces (TBD)
- Write Pos, Norm, Indices to OpenGL VBOs
- VBO can then be re-used for multiple purposes

Directly Tessellating NURBS Surfaces



Tessellating NURBS Surfaces



CUDA Tessellation

- Input is an array of NURBS surfaces in device mem
 - Surface: CV's, UV knots, UV degree, boundary cond, ...
- One NURBS surface handled per CUDA block
 - 1) Compute tessellation levels (separate kernel)
 - 2) Pre-compute polynomial coefficients on knot spans
 - 3) Compute custom (u,v) coordinates per vertex
 - 4) Compute vertex position, normal at each (u,v)
 - 5) Index through all quads, compute triangle indices

1) Compute Tessellation Levels

- Use CUDA Kernel to compute edge tessellation levels
- Simple formula for this demo
 - $\text{tessLevel} = C * (\sum \text{length}(\text{CP}[i+1] - \text{CP}[i])) / \text{distanceToCamera}$
 - $i = 0 \text{ to } (\#\text{CPs on edge}) - 1$
 - C is user-defined constant
- Generates relatively constant triangle size on screen
- All vertices for all patches tessellated into one large VBO
- Must also compute unique pointers into VBO per patch
 - $\text{vertexIndex} = \text{atomicAdd}(\text{nTotalVertices}, \text{nVertsInSurface})$
 - $\text{patch} \rightarrow \text{vertexVBOptr} = \text{VBOStart} + \text{vertexIndex} * \text{sizeof}(\text{float4})$

2) Pre-compute basis polynomial coefficients, store in shared memory

- NURBS basis functions expanded in polynomial series

$$B_{i,n}(t) = \sum_{k=0}^n C_{i,n,k}(t) \cdot t^k$$

- Pre-compute coefficients per knot span (indep of t)

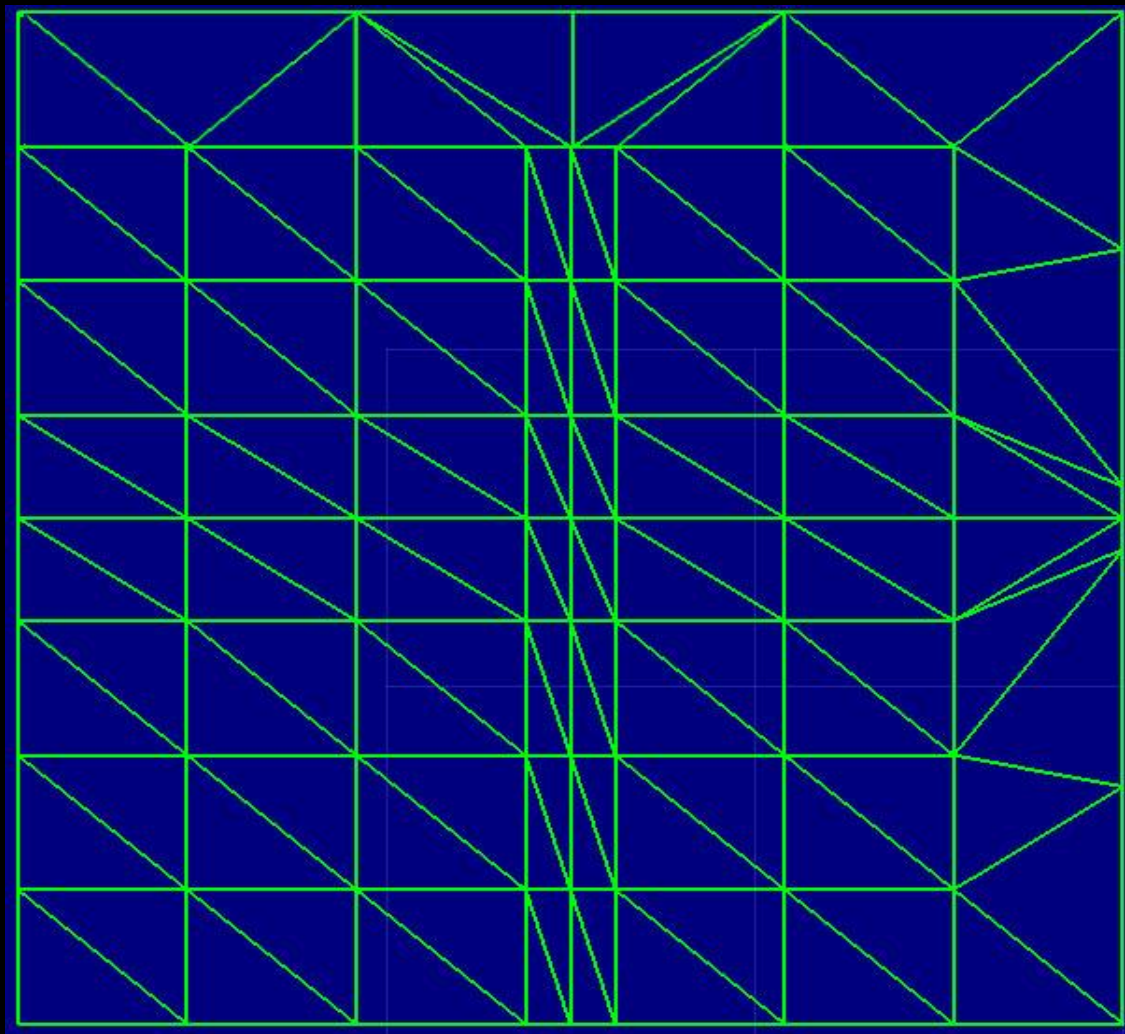
$$C_{i,0,0}(t) = B_{i,0}(t)$$

$$C_{i,n,0}(t) = \frac{t_{i+n+1}}{t_{i+n+1} - t_{i+1}} C_{i+1,n-1,0}(t) - \frac{t_i}{t_{i+n} - t_i} C_{i,n-1,0}(t)$$

$$C_{i,n,n}(t) = \frac{1}{t_{i+n} - t_i} C_{i,n-1,n-1}(t) - \frac{1}{t_{i+n+1} - t_{i+1}} C_{i+1,n-1,n-1}(t)$$

$$\forall k \in \{1..n-1\}, C_{i,n,k}(t) = \frac{C_{i,n-1,k-1}(t) - t_i \cdot C_{i,n-1,k}(t)}{t_{i+n} - t_i} - \frac{C_{i+1,n-1,k-1}(t) - t_{i+n+1} \cdot C_{i+1,n-1,k}(t)}{t_{i+n+1} - t_{i+1}}$$

3) Fractional, Symmetric U,V Tessellation



3) Compute U,V Tessellation Pattern

- Loop over all verts, increment i by blockDim.x
 $\text{idx} = i + \text{threadIdx.x};$
 $\text{idxU} = \text{idx} \% \text{nVertsU};$
 $\text{idxV} = \text{idx} / \text{nVertsU};$
- Compute symmetric, fractional UV tessellation
 $u = u0 + (\text{float})\text{idxU} * w;$ $\text{idxU} < \text{ceilTessU} * 0.5f - \text{EPSILON}$
 $u = un - \text{EPSILON} - (\text{ceilTessU} - (\text{float})\text{idxU}) * w;$ $\text{idxU} > \text{ceilTessU} * 0.5f + \text{EPSILON}$
 $u = u = u0 + 0.5f * (un - u0);$ otherwise
- For differing edge tess, make some vertices redundant:
 $\text{idxU0} = (\text{int})((\text{ceil}(\text{edgeTess}[0]) / \text{ceil}(\text{tessFactorU})) * (\text{float})\text{idxU});$
 //compute u with idxU0 as above

4) Compute Vertex Positions and Normals

- Use pre-computed polynomial coefficients
- Compute vertex positions

$$S(u, v) = \frac{\sum_{i=0}^p \sum_{j=0}^q B_{i,m}(u) \cdot B_{j,n}(v) \cdot P_{i,j} \cdot w_{i,j}}{\sum_{i=0}^p \sum_{j=0}^q B_{i,m}(u) \cdot B_{j,n}(v) \cdot w_{i,j}}$$

$$B_{i,n}(t) = \sum_{k=0}^n C_{i,n,k}(t) \cdot t^k$$

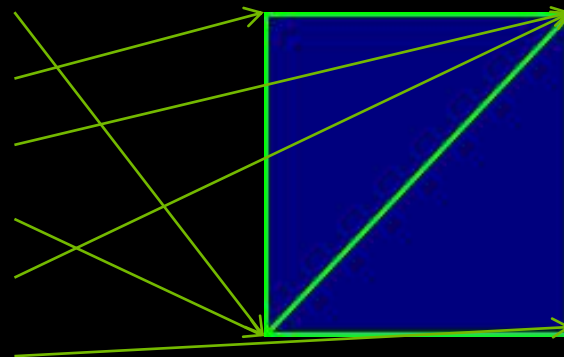
$$\frac{dB_{i,n}(t)}{dt} = \sum_{k=1}^n k \cdot C_{i,n,k}(t) \cdot t^{k-1}$$

- Use Horner's rule to efficiently evaluate polynomials
- Compute U,V tangent vectors -> vertex normals
 - Use polynomial coefficients to compute derivatives

5) Compute Triangulation Indices

- Compute # of quads in surface
- Loop over all quads, increment i by blockIdx.x
- $\text{idx} = i + \text{threadIdx.x}$
- Compute $\text{idxU} = \text{idx} \% \text{nQuadsU}$, $\text{idxV} = \text{idx} / \text{nQuadsU}$
- Compute indices for triangles

```
index[idx*6] = offset + idxU + idxV*nVerticesU;  
index[idx*6+1] = offset + idxU + (idxV+1)*nVerticesU;  
index[idx*6+2] = offset + idxU+1 + (idxV+1)*nVerticesU;  
index[idx*6+3] = offset + idxU + idxV*nVerticesU;  
index[idx*6+4] = offset + idxU+1 + (idxV+1)*nVerticesU;  
index[idx*6+5] = offset + idxU+1 + idxV*nVerticesU;
```



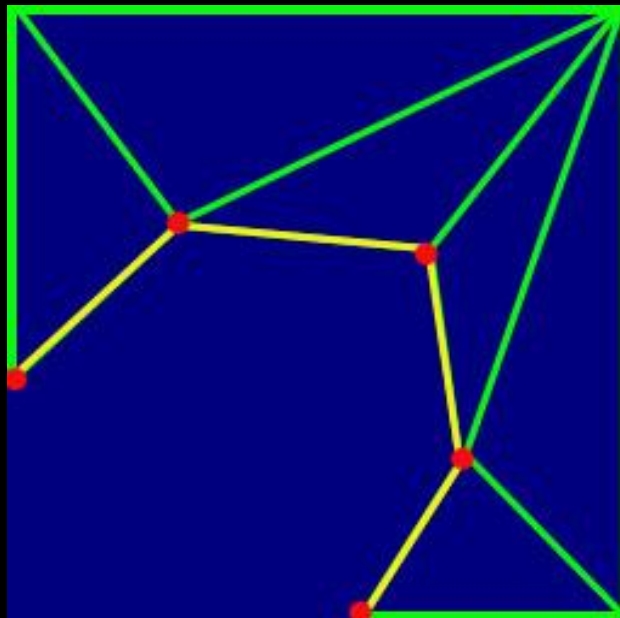
Results (438 surf, 3.5M tri in 19.2 ms)



Performance Optimizations

- Set `cudaFuncSetCacheConfig(cudaFuncCachePreferShared)`
 - Increases the shared memory available to 48 KB
 - Increases occupancy by allowing more blocks to run simultaneously
- Launch with `blockDim (128,1,1)`
 - Empirically tested to be the most efficient configuration
- Set max regcount to 32
 - Spills more registers to L1
 - Increases occupancy

Trimmed Surfaces



Questions?

- Feel free to contact me:
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Samples will be posted after GTC

