In this chapter we describe a new technique for computing diffuse light transfer and show how it can be used to compute global illumination for animated scenes. Our technique is efficient enough when implemented on a fast GPU to calculate ambient occlusion and indirect lighting data on the fly for each rendered frame. It does not have the limitations of precomputed radiance transfer (PRT) or precomputed ambient occlusion techniques, which are limited to rigid objects that do not move relative to one another (Sloan 2002). Figure 14-1 illustrates how ambient occlusion and indirect lighting enhance environment lighting.

Our technique works by treating polygon meshes as a set of surface elements that can emit, transmit, or reflect light and that can shadow each other. This method is so efficient because it works without calculating the visibility of one element to another. Instead, it uses a much simpler and faster technique that uses shadowing to account for occluding (blocking) geometry.

14.1 Surface Elements

The first step in our algorithm is to convert the polygonal data to surface elements to make it easy to calculate how much one part of a surface shadows or illuminates another. Figure 14-2 illustrates the basic concept. We define a surface element as an oriented disk with a position, normal, and area. An element has a front face and a back
face. Light is emitted and reflected from the front-facing side. Light is transmitted and shadows are cast from the back. We create one element per vertex. Assuming that the vertices are defined as a position and normal already, we just need to calculate the area of each element. We calculate the area at a vertex as the sum of one-third of the area of the triangles that share the vertex (or one-fourth of the area for quads). Heron’s formula for the area of a triangle with sides of length \(a\), \(b\), and \(c\) is:

\[
\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}
\]

where \(s = \frac{a+b+c}{2}\) is the semi-perimeter of the triangle.
where \( s \) is half the perimeter of the triangle: \((a + b + c)/2\).

We store element data (position, normal, and area) in a texture map because we will be using a fragment program (that is, a pixel shader) to do all the ambient occlusion calculations. Assuming that vertex positions and normals will change for each frame, we need to be able to change the values in the texture map quickly. One option is to keep vertex data in a texture map from the start and to do all the animation and transformation from object space to eye (or world) space with fragment programs instead of vertex programs as usual. We can use render-to-vertex array to create the array of vertices to be sent down the regular pipeline, and then use a simple pass-through vertex shader. Another, less efficient option is to do the animation and transformation on the CPU and load a texture with the vertex data each frame.

### 14.2 Ambient Occlusion

Ambient occlusion is a useful technique for adding shadowing to diffuse objects lit with environment lighting. Without shadows, diffuse objects lit from many directions look flat and unrealistic. Ambient occlusion provides soft shadows by darkening surfaces that are partially visible to the environment. It involves calculating the accessibility value, which is the percentage of the hemisphere above each surface point not occluded by geometry (Landis 2002). In addition to accessibility, it is also useful to calculate the direction of least occlusion, commonly known as the **bent normal**. The bent normal is used in place of the regular normal when shading the surface for more accurate environment lighting.

We can calculate the accessibility value at each element as 1 minus the amount by which all the other elements shadow the element. We refer to the element that is shadowed as the **receiver** and to the element that casts the shadow as the **emitter**. We use an approximation based on the solid angle of an oriented disk to calculate the amount by which an emitter element shadows a receiver element. Given that \( A \) is the area of the emitter, the amount of shadow can be approximated by:

\[
1 - \frac{r \cos \theta_E \max(1, 4 \cos \theta_R)}{\sqrt{\frac{A}{\pi} + r^2}} \left( 1 - \frac{r}{\sqrt{\frac{A}{2} + r^2}} \right) \cos \theta_E \times \min(1, 4 \times \cos \theta_R).
\]

*Equation 14-1. Shadow Approximation*
As illustrated in Figure 14-3, $\theta_E$ is the angle between the emitter’s normal and the vector from the emitter to the receiver. $\theta_R$ is the corresponding angle for the receiver element. The $\max(1, 4 \times \cos \theta_R)$ term is added to the disk solid angle formula to ignore emitters that do not lie in the hemisphere above the receiver without causing rendering artifacts for elements that lie near the horizon.

Here is the fragment program function to approximate element-to-element solid angle:

```plaintext
float SolidAngle(float3 v, float d2, float3 receiverNormal, float3 emitterNormal, float emitterArea)
{
    // we assume that emitterArea has already been divided by PI
    return (1 - rsqrt(emitterArea/d2 + 1)) *
           saturate(dot(emitterNormal, v)) *
           saturate(4 * dot(receiverNormal, v));
}
```

**Figure 14-3.** The Relationship Between Receiver and Emitter Elements
Receiver element R receives light or shadow from emitter E with r as the distance between the centers of the two elements.

### 14.2.1 The Multipass Shadowing Algorithm

We calculate the accessibility values in two passes. In the first pass, we approximate the accessibility for each element by summing the solid angles subtended by every other
element and subtracting the result from 1. After the first pass, some elements will generally be too dark because other elements that are in shadow are themselves casting shadows. So we use a second pass to do the same calculation but this time to multiply each form factor by the emitter element’s accessibility from the last pass. The effect is that elements that are in shadow will cast fewer shadows on other elements, as illustrated in Figure 14-4. After the second pass, we have removed any double shadowing. However, surfaces that are triple shadowed or more will end up being too light. We can use more passes to get a better approximation, but we can approximate the same answer by using a weighted average of the combined results of the first and second passes. Figure 14-5 shows the results after each pass, as well as a ray-traced solution for comparison. The bent normal calculation is done during the second pass. We compute the bent normal by first multiplying the normalized vector between elements and the form factor. Then we subtract this result from the original element normal.

Figure 14-4. Correcting for Occlusion by Overlapping Objects
Left: Elements A and B correctly shadow C after the first pass. Middle: In this arrangement, B casts too much shadow on C. Right: B’s shadow is adjusted by the second pass to shadow C properly.

Figure 14-5. Comparing Models Rendered with Our Technique to Reference Images
Model rendered with ambient occlusion accessibility value calculated after (a) one pass, (b) two passes, and (c) three passes of the shader. (d) Image made by tracing 200 random rays per vertex for comparison.
We calculate the occlusion result by rendering a single quad (or two triangles) so that one pixel is rendered for each surface element. The shader calculates the amount of shadow received at each element and writes it as the alpha component of the color of the pixel. The results are rendered to a texture map so the second pass can be performed with another render. In this pass, the bent normal is calculated and written as the RGB value of the color with a new shadow value that is written in the alpha component.

14.2.2 Improving Performance

Even though the element-to-element shadow calculation is very fast (a GeForce 6800 can do 150 million of these calculations per second), we need to improve our algorithm to work on more than a couple of thousand elements in real time. We can reduce the amount of work by using simplified geometry for distant surfaces. This approach works well for diffuse lighting environments, because the shadows are so soft that those cast by details in distant geometry are not visible. Fortunately, because we do not use the polygons themselves in our technique, we can create surface elements to represent simplified geometry without needing to create alternate polygonal models. We simply group elements whose vertices are neighbors in the original mesh and represent them with a single, larger element. We can do the same thing with the larger elements, creating fewer and even larger elements, forming a hierarchy. Now instead of traversing every single element for each pixel we render, we traverse the hierarchy of elements. If the receiver element is far enough away from the emitter—say, twice the radius of the emitter—we use it for our calculation. Only if the receiver is close to an emitter do we need to traverse its children (if it has any). See Figure 14-6. By traversing a hierarchy in this way, we can improve the performance of our algorithm from $O(n^2)$ to $O(n \log n)$ in practice. The chart in Figure 14-7 shows that the performance per vertex stays consistent when the number of vertices in the hierarchy increases.

![Figure 14-6. Hierarchical Elements](image)

*Elements are traversed in a hierarchy; child elements are traversed only if necessary.*
We use the `createParentElements` and `updateParentElements` shaders, found on the book’s CD, to create the element data for nonleaf elements in the hierarchy. We use `createParentElements` to calculate the position, normal, and area values when they change. We use `updateParentElements` before each shader pass that needs the result of the last pass. It calculates a sum of child element results weighted by the child’s area. Both shaders are called for each level in the hierarchy, starting at parents of leaf nodes and moving up, so they can calculate their results by looking only at the direct descendants of an element. It is worth noting that the area of most elements varies little, if at all, even for nonrigid objects; therefore, the area does not have to be recalculated for each frame.

The ambient occlusion fragment shader appears in Listing 14-1.

**Listing 14-1. Ambient Occlusion Shader**

```csh
float4 AmbientOcclusion(
    float4 position : WPOS,
    float4 normOffset : TEX1,
```

![Figure 14-7. Ambient Occlusion Shader Performance for Meshes of Different Densities](image)
Listing 14-1 (continued). Ambient Occlusion Shader

```glsl
uniform samplerRECT lastResultMap : TEXUNIT0,
uniform samplerRECT positionMap : TEXUNIT1,
uniform samplerRECT elementNormalMap : TEXUNIT2,
uniform samplerRECT indexMap : TEXUNIT3) : COL
{
    float eArea;       // emitter area
    float4 ePosition;  // emitter position
    float4 eNormal;    // emitter normal
    float3 rPosition = texRECT(positionMap, position.xy).xyz;
    float3 rNormal = texRECT(elementNormalMap, position.xy).xyz;
    float3 v;          // vector from receiver to emitter
    float total = 0;    // used to calculate accessibility
    float4 eIndex = float2(0.5, 0.5); // index of current emitter
    float value,
    float d2;          // distance from receiver to emitter squared

    while (emitterIndex.x != 0) { // while not finished traversal
        ePosition = texRECT(positionMap, emitterIndex.xy);
        eNormal = texRECT(elementNormalMap, emitterIndex.xy);
        eArea = emitterNormal.w;
        eIndex = texRECT(indexMap, emitterIndex.xy); // get next index
        v = ePosition.xyz - rPosition; // vector to emitter
        d2 = dot(v, v) + 1e-16; // calc distance squared, avoid 0
        // is receiver close to parent element?
        if (d2 < -eArea) { // (parents have negative area)
            eIndex.xy = eIndex.zw; // go down hierarchy
            emitterArea = 0; // ignore this element
        }
        v *= rsqrt(d2); // normalize v
        value = SolidAngle(v, d2, rNormal, eNormal.xyz, abs(eArea)) *
        texRECT(resultMap, eIndex.xy).w; // modulate by last result
        bentNormal -= value * v; // update bent normal
        total += value;
    }

    if (!lastPass) // only need bent normal for last pass
        return 1 - total; // return accessibility only
    return float4(normalize(bentNormal), 1 - total);
}
```
14.3 Indirect Lighting and Area Lights

We can add an extra level of realism to rendered images by adding indirect lighting caused by light reflecting off diffuse surfaces (Tabellion 2004). We can add a single bounce of indirect light using a slight variation of the ambient occlusion shader. We replace the solid angle function with a disk-to-disk radiance transfer function. We use one pass of the shader to transfer the reflected or emitted light and two passes to shadow the light.

For indirect lighting, first we need to calculate the amount of light to reflect off the front face of each surface element. If the reflected light comes from environment lighting, then we compute the ambient occlusion data first and use it to compute the environment light that reaches each vertex. If we are using direct lighting from point or directional lights, we compute the light at each element just as if we are shading the surface, including shadow mapping. We can also do both environment lighting and direct lighting and sum the two results. We then multiply the light values by the color of the surface element, so that red surfaces reflect red, yellow surfaces reflect yellow, and so on. Area lights are handled just like light-reflective diffuse surfaces except that they are initialized with a light value to emit.

Here is the fragment program function to calculate element-to-element radiance transfer:

```cpp
float FormFactor(float3 d, float d2, float3 receiverNormal, float3 emitterNormal, float emitterArea) {
    // assume that emitterArea has been divided by PI
    return emitterArea * saturate(dot(emitterNormal, d)) * saturate(dot(receiverNormal, d)) / (d2 + emitterArea);
}
```

Equation 14-2. Disk-to-Disk Form Factor Approximation

\[
\frac{A \cos \theta_E \cos \theta_R}{\pi r^2 + A}
\]

Here is the fragment program function to calculate element-to-element radiance transfer:

```cpp
float FormFactor(float3 v, float d2, float3 receiverNormal, float3 emitterNormal, float emitterArea) {
    // assume that emitterArea has been divided by PI
    return emitterArea * saturate(dot(emitterNormal, v)) * saturate(dot(receiverNormal, v)) / (d2 + emitterArea);
}
```

We calculate the amount of light transferred from one surface element to another using the geometric term of the disk-to-disk form factor given in Equation 14-2. We leave off the visibility factor, which takes into account blocking (occluding) geometry. Instead we use a shadowing technique like the one we used for calculating ambient occlusion—only this time we use the same form factor that we used to transfer the light. Also, we multiply the shadowing element’s form factor by the three-component light value instead of a single-component accessibility value.
We now run one pass of our radiance-transfer shader to calculate the maximum amount of reflected or emitted light that can reach any element. Then we run a shadow pass that subtracts from the total light at each element based on how much light reaches the shadowing elements. Just as with ambient occlusion, we can run another pass to improve the lighting by removing double shadowing. Figure 14-8 shows a scene lit with direct lighting plus one and two bounces of indirect lighting.

**Figure 14-8.** Combining Direct and Indirect Lighting
*Top, left to right:* Scene lit with direct lighting, with direct lighting plus one bounce of indirect lighting, and with direct lighting plus two bounces of indirect lighting. *Bottom, left to right:* Indirect lighting after one pass, after two passes (one bounce), and after two bounces (four passes total).

### 14.4 Conclusion

Global illumination techniques such as ambient occlusion and indirect lighting greatly enhance the quality of rendered diffuse surfaces. We have presented a unique technique for calculating light transfer to and from diffuse surfaces using the GPU. This technique is suitable for implementing various global illumination effects in dynamic scenes with deformable geometry.
14.5 References and Further Reading


