Efficient Graph Matching and Coloring on the GPU

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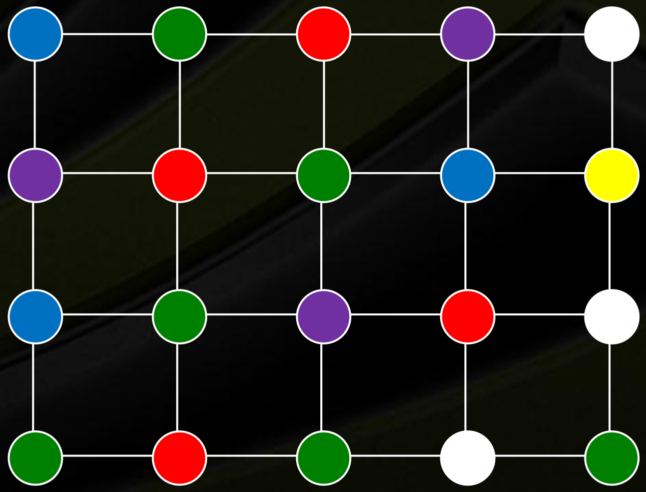
Key Idea

- Cost model: count global syncs
- Reasoning:
  - One kernel invocation = One global sync
    1) Read graph data
    2) Compute something,
    3) Write results
    4) Wait for all threads to finish (sync)
  - Assume “read graph data” and “wait” (sync) dominate
- Model is too crude today, but leads to algorithms that scale to future trends (and bigger machines)
- Reducing kernel launches generally improves perf
- Conclusion: want coloring and matching algorithms requiring fewest number of kernel launches

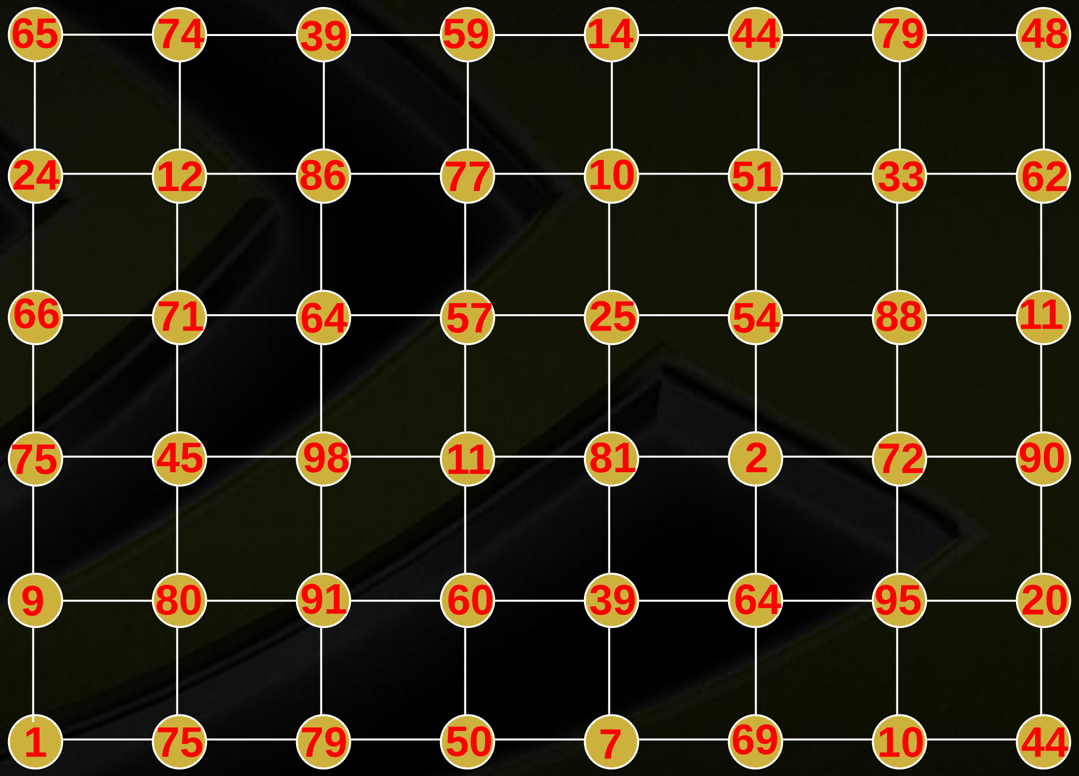
Graph Coloring

- Assignment of “color” (integer) to vertices, with no two adjacent vertices the same color
- Each color forms independent set (conflict-free)
  - reveals parallelism inherent in graph topology
- “inexact” coloring is often ok
- Our focus: fast, cheap, non-optimal colorings



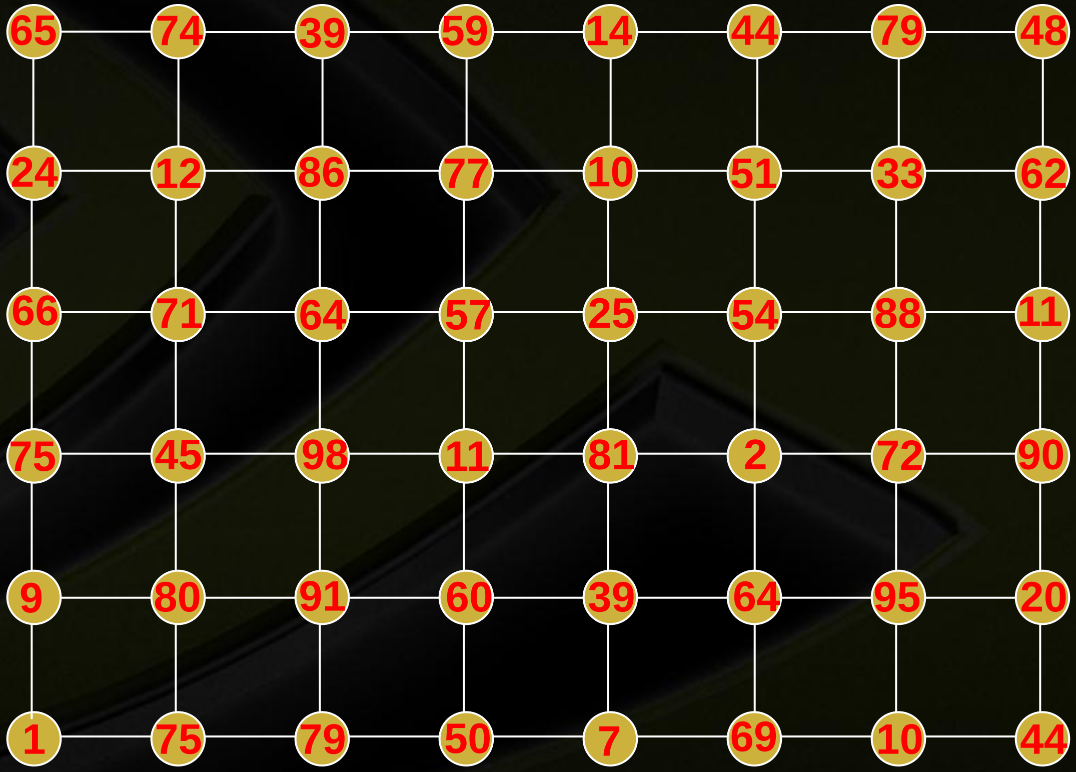
Parallel Graph Coloring – Luby-Jones

- **Parallel graph coloring algorithm of Luby / Jones-Plassman**



Parallel Graph Coloring – Luby-Jones

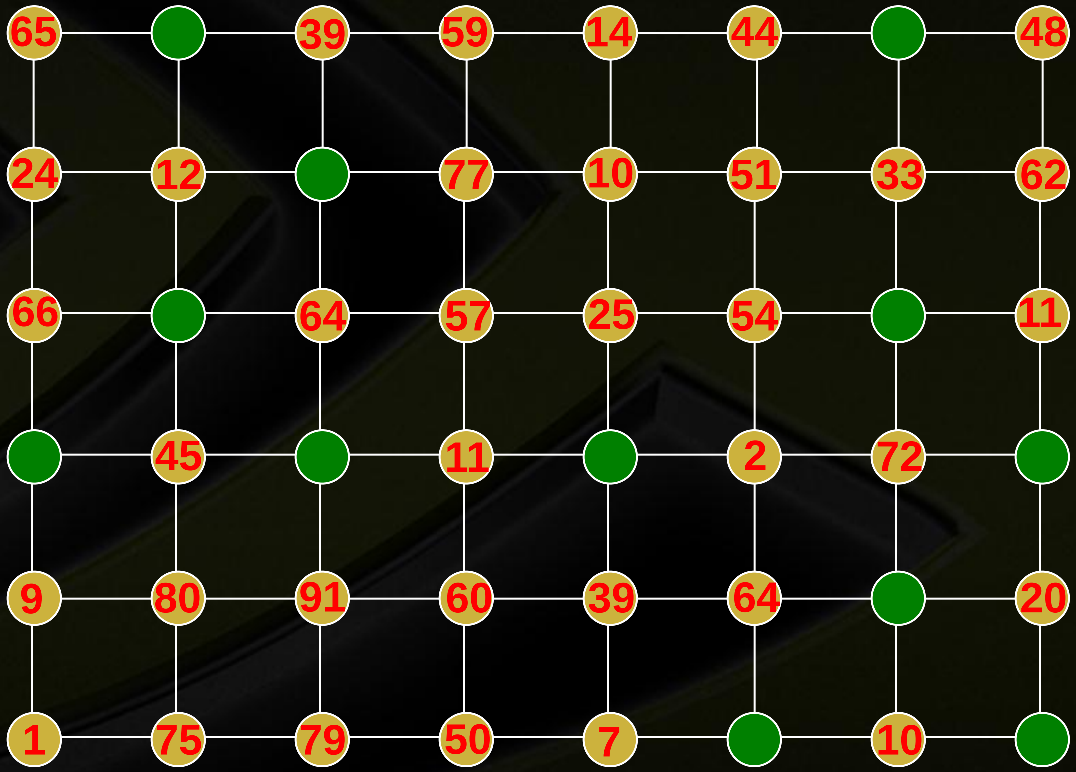
- Classic approach: compute array of random numbers
- First optimization: compute a hash function of vertex index on the fly
- Vertex can compute hash number of its neighbors’ indices
- Trades bandwidth for compute, skip kernel to assign random numbers



65	74	39	59	14	44	79	48
24	12	86	77	10	51	33	62
66	71	64	57	25	54	88	11
75	45	98	11	81	2	72	90
9	80	91	60	39	64	95	20
1	75	79	50	7	69	10	44

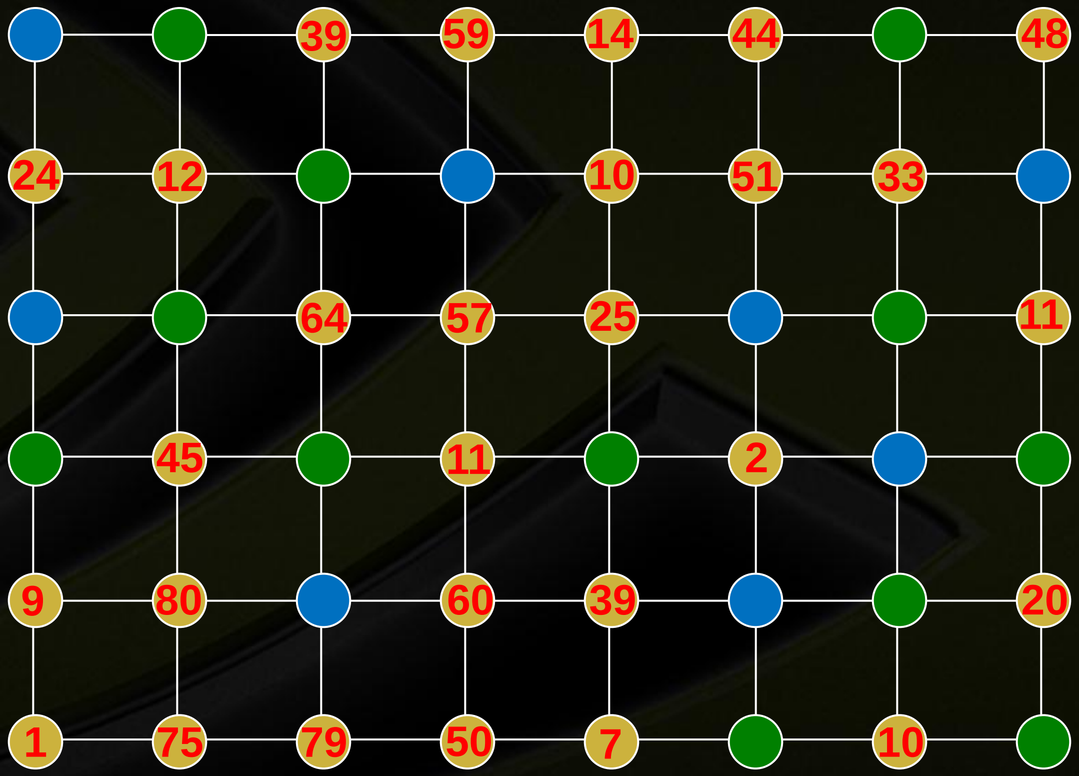
Parallel Graph Coloring – Luby-Jones

- Round 1: Each vertex checks if local maximum
- => Adjacent vertices can’t both be local maxima
- If max, color=green.



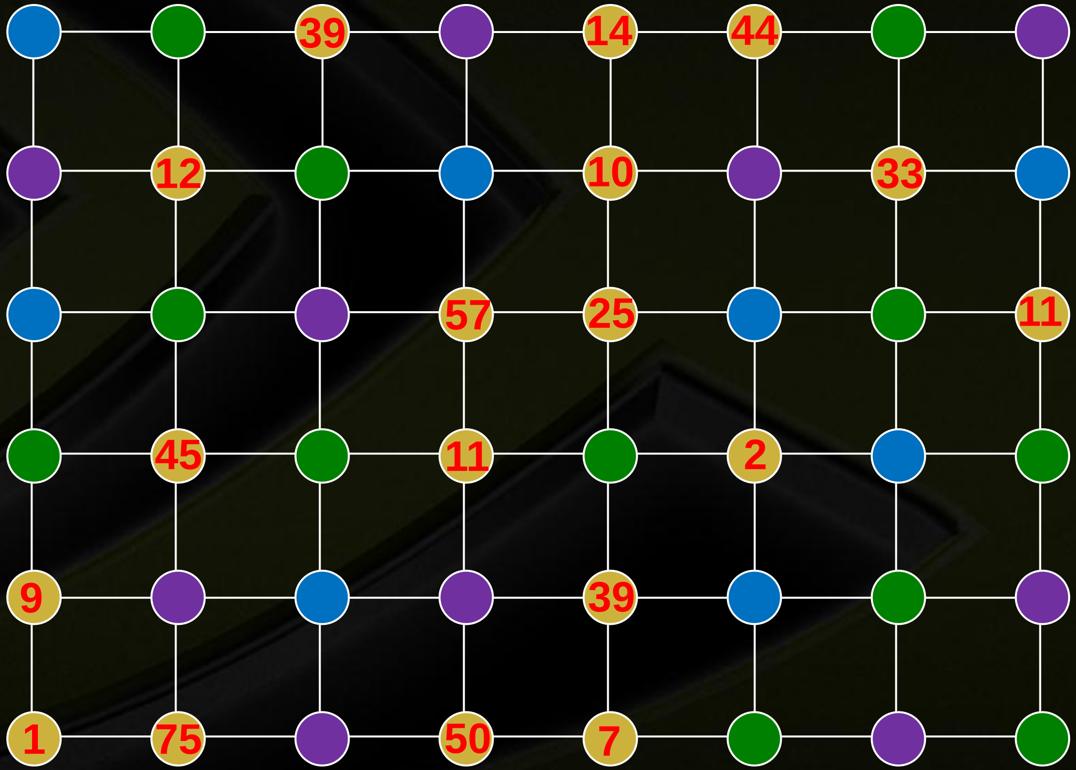
Parallel Graph Coloring – Luby-Jones

- Round 2: Each vertex checks if local maximum, ignoring green
- If max, color=blue



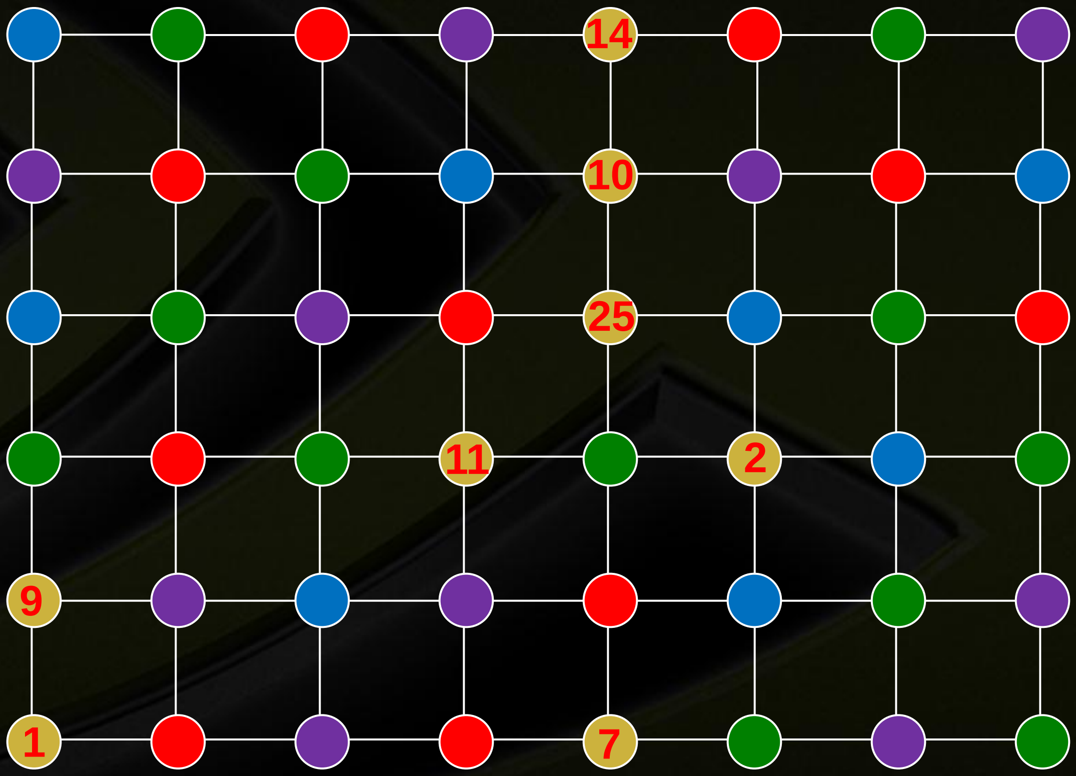
Parallel Graph Coloring – Luby-Jones

- **Round 3: Each vertex checks if local maximum, ignoring colored nbrs**
- **If max, color=purple**



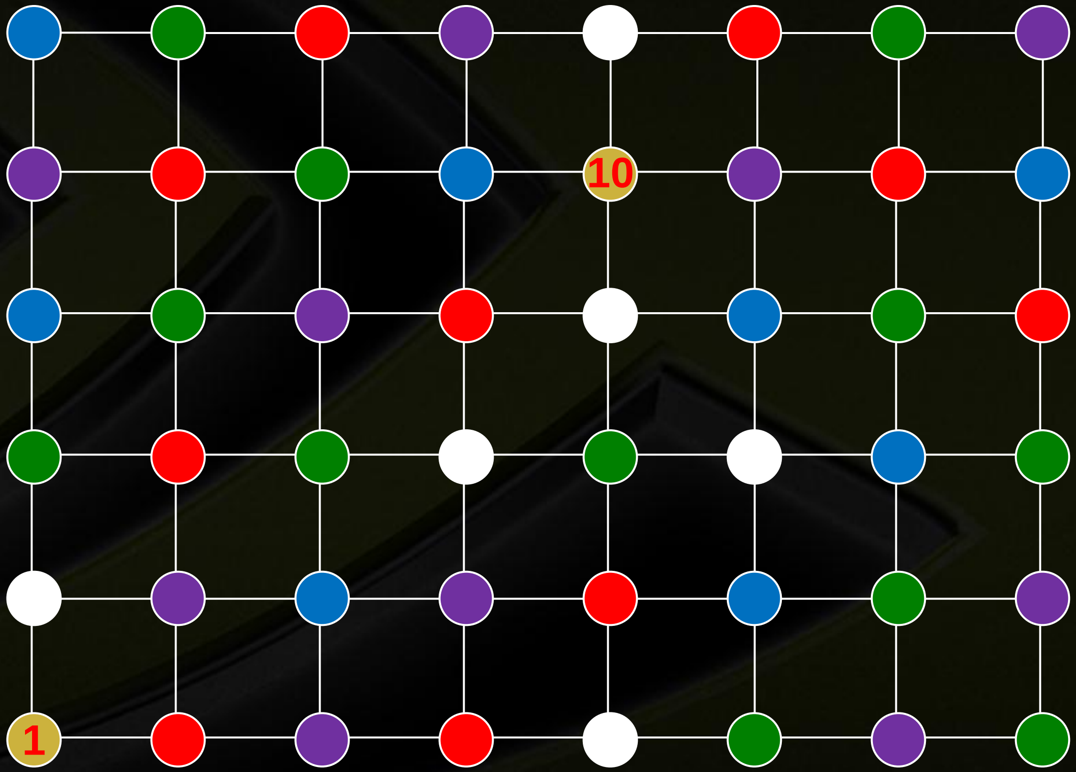
Parallel Graph Coloring – Luby-Jones

- Round 4: Each vertex checks if local maximum, ignoring colored nbrs
- If max, color=red



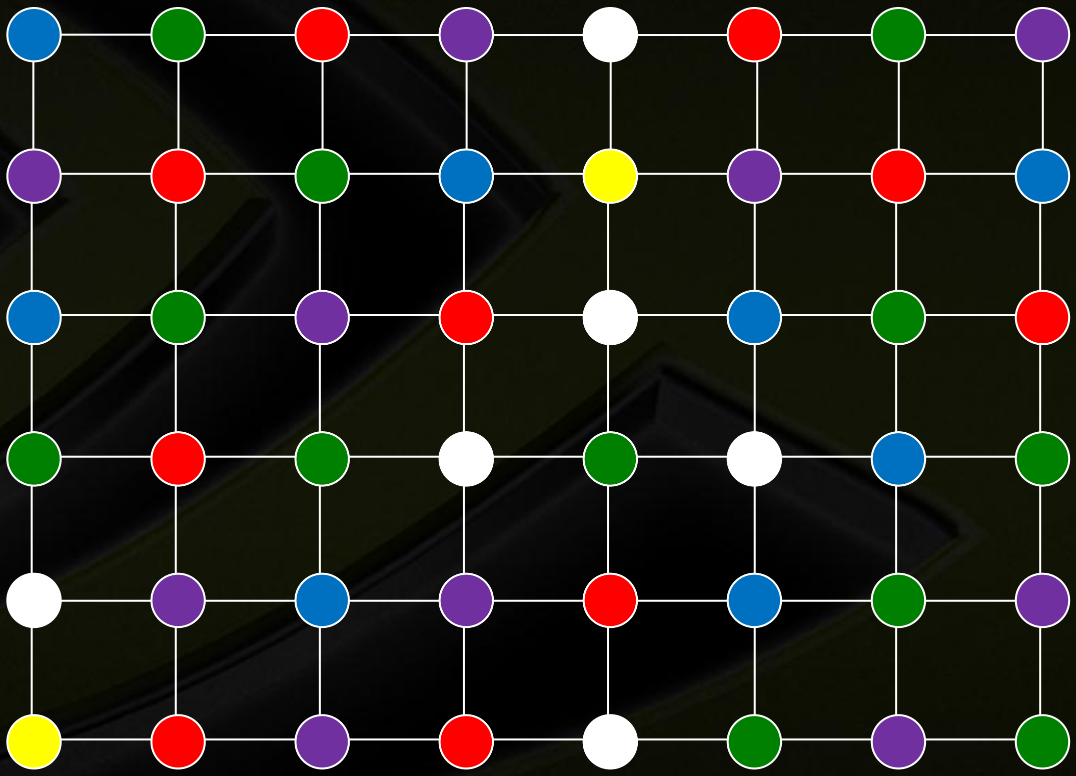
Parallel Graph Coloring – Luby-Jones

- Round 5: Each vertex checks if local maximum, ignoring colored nbrs
- If max, color=white



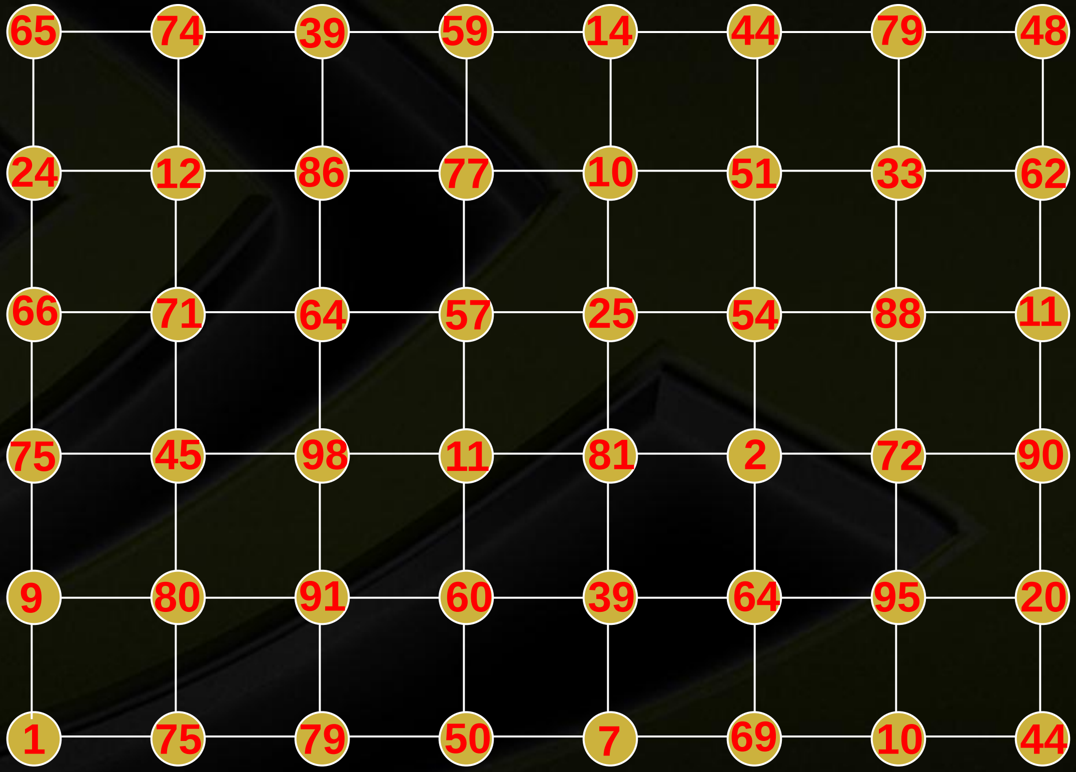
Parallel Graph Coloring – Luby-Jones

- Round 6: Each vertex checks if local maximum, ignoring colored nbrs
- If max, color=yellow
- Completes in 6 rounds



Parallel Graph Coloring – Min-Max

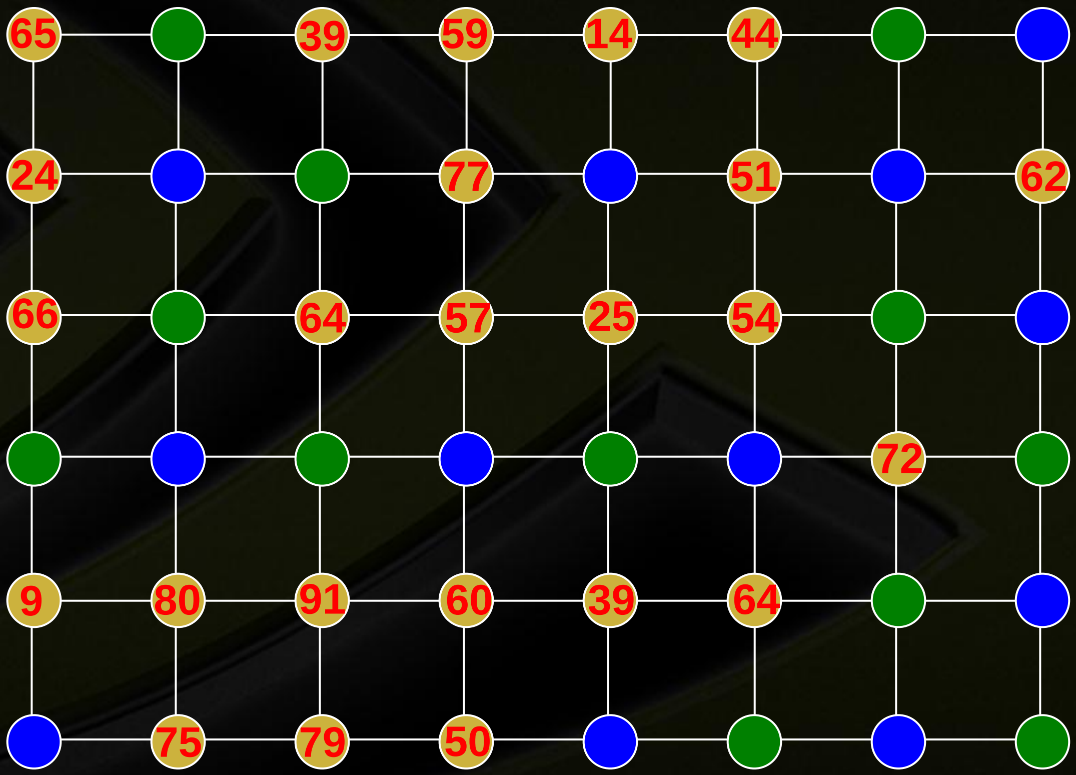
- **Realization: Local min and local max are both independent sets**
- **They are disjoint => can produce 2 colors per iteration**



65 74 39 59 14 44 79 48
24 12 86 77 10 51 33 62
66 71 64 57 25 54 88 11
75 45 98 11 81 2 72 90
9 80 91 60 39 64 95 20
1 75 79 50 7 69 10 44

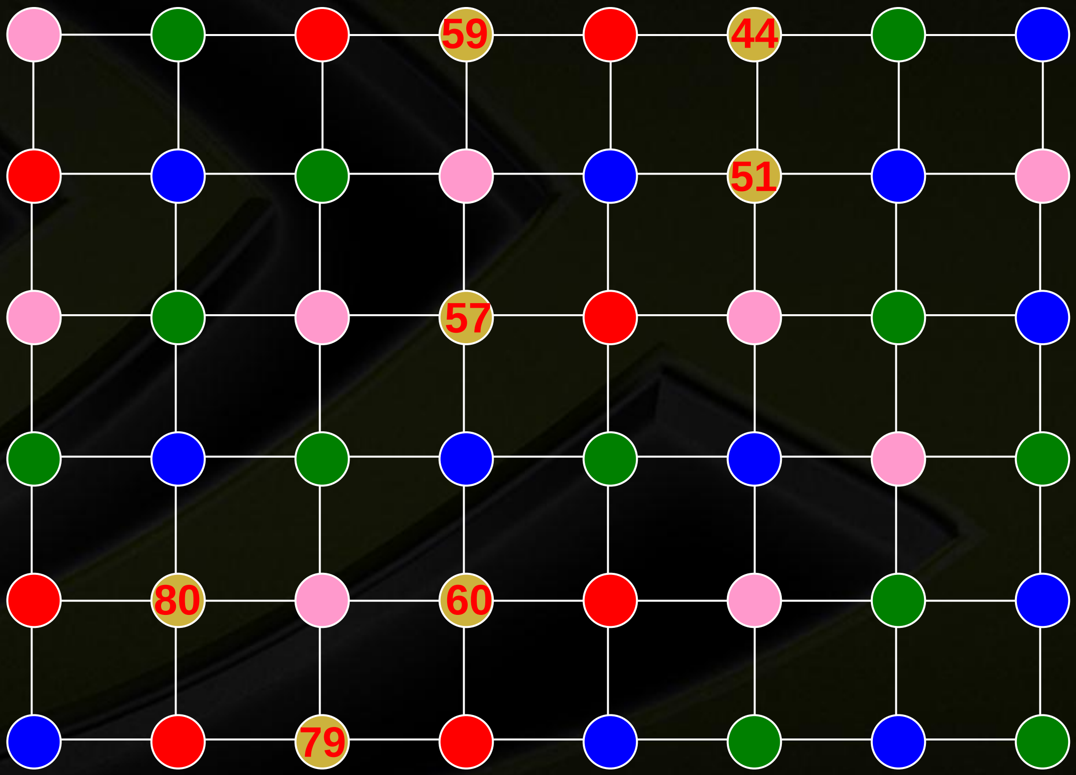
Parallel Graph Coloring – Min/Max

- Round 1: Each vertex checks if it’s a local maximum or minimum.
- If max, color=blue. If min, color=green



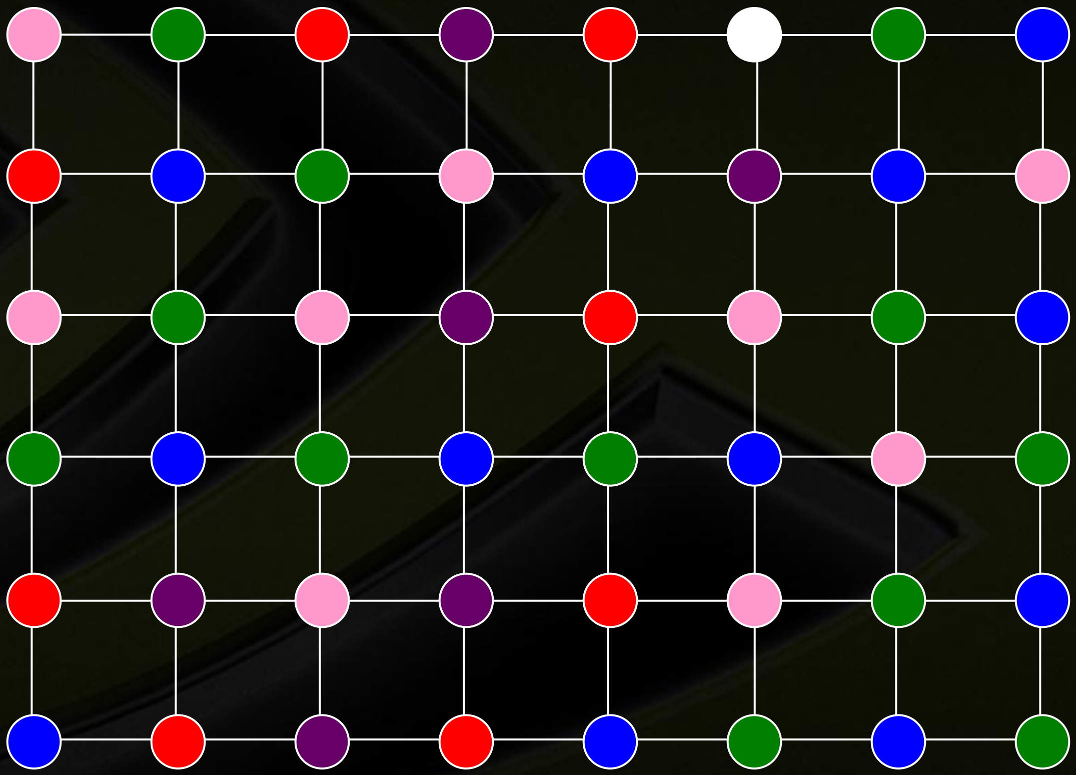
Parallel Graph Coloring – Min/Max

- Round 2: Each vertex checks if it’s a local maximum or minimum.
- If max, color=pink. If min, color=red



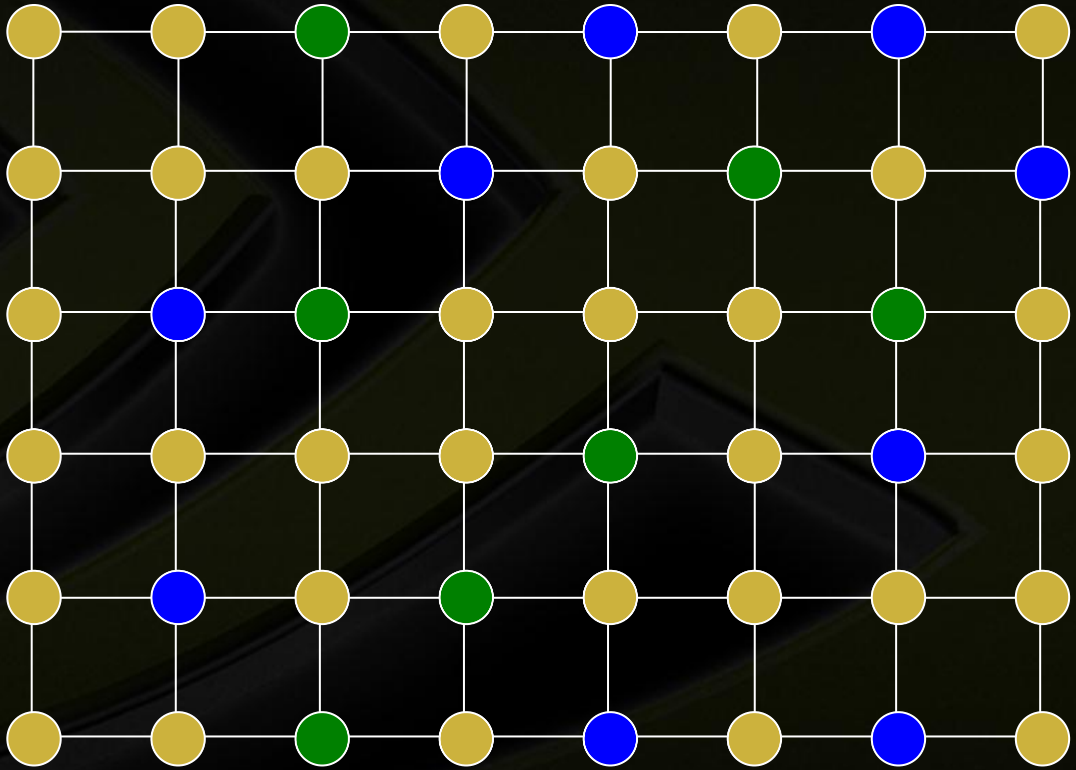
Parallel Graph Coloring – Min/Max

- Round 3: Each vertex checks if it’s a local maximum or minimum.
- If max, color=purple. If min, color=white
- Improvement: 3 rounds versus 6



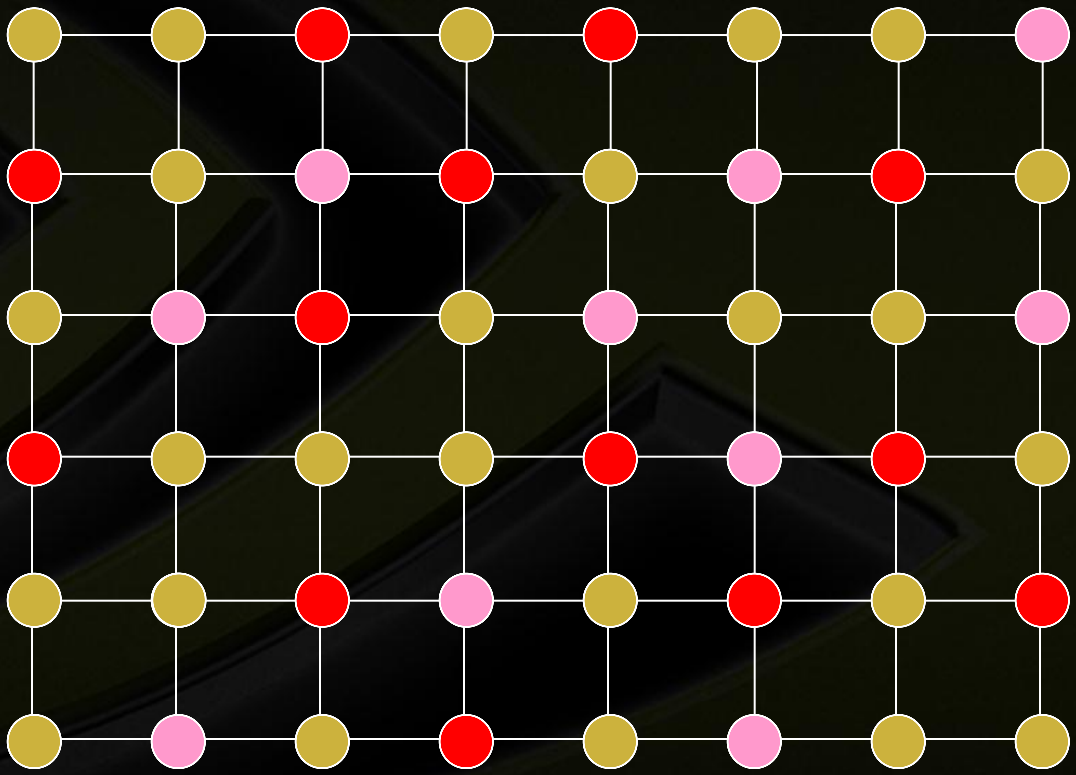
Parallel Graph Coloring – Multi-Hash

- Use multiple hash functions to obtain multiple 2-coloring of the graph
- Hash function 1:



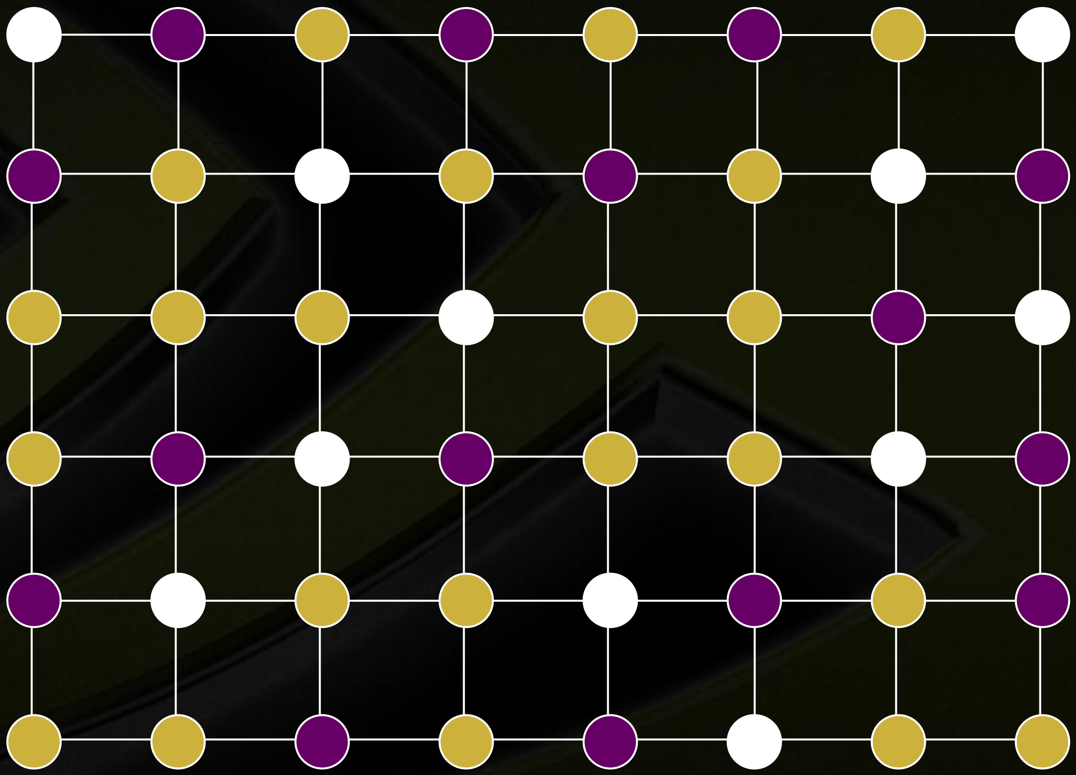
Parallel Graph Coloring – Multi-Hash

- Use multiple hash functions to obtain multiple 2-coloring of the graph
- Hash function 2:



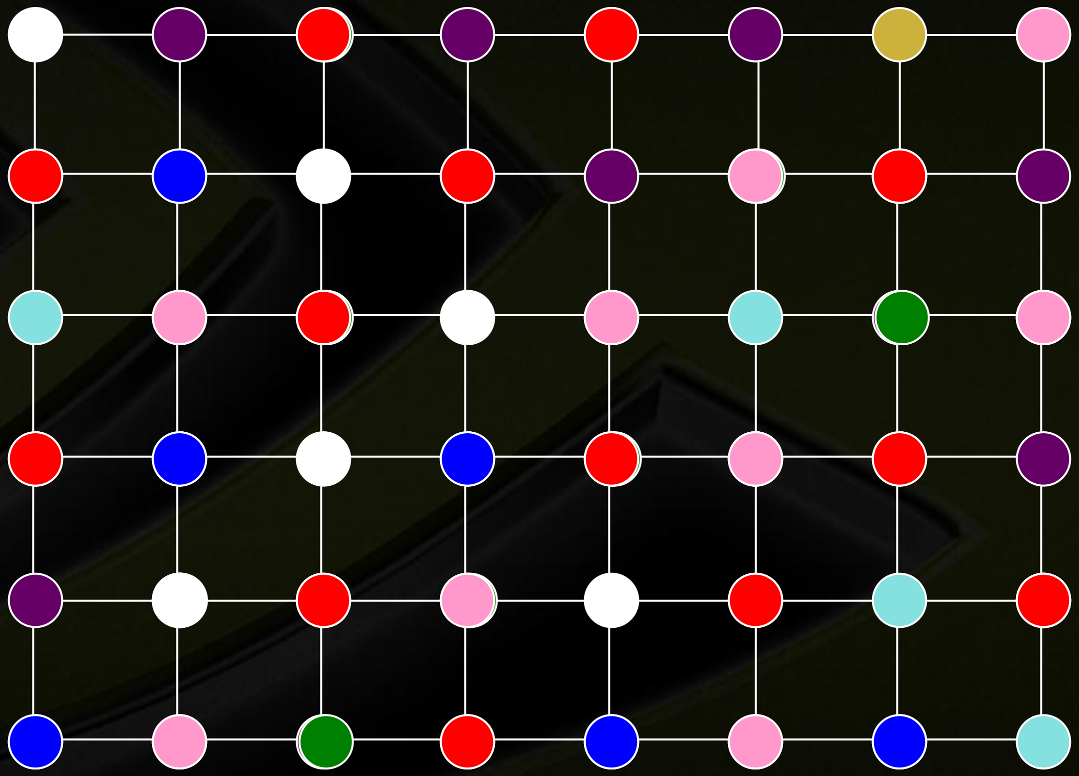
Parallel Graph Coloring – Multi-Hash

- Use multiple hash functions to obtain multiple 2-coloring of the graph
- Hash function 3:

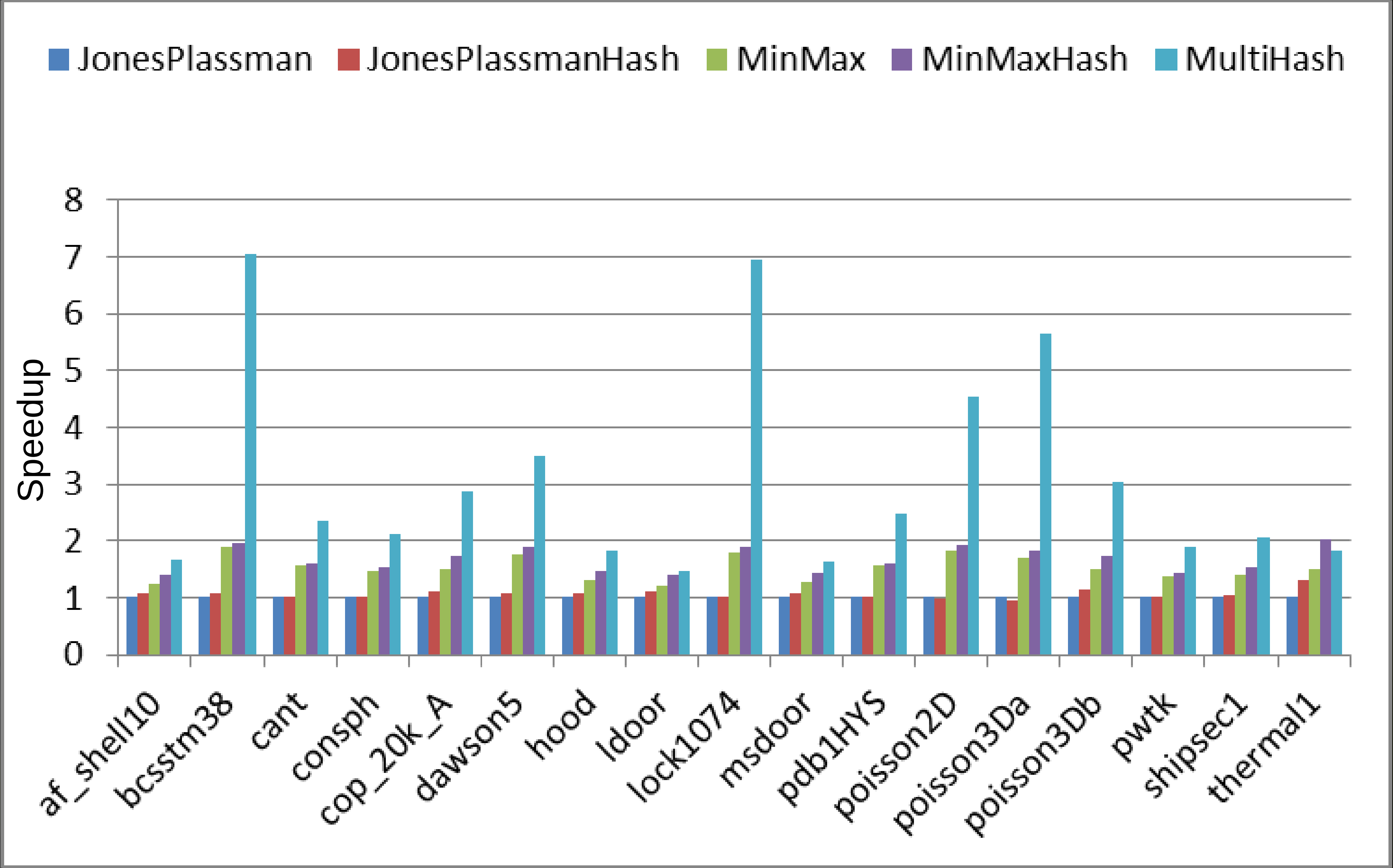


Parallel Graph Coloring – Multi-Hash

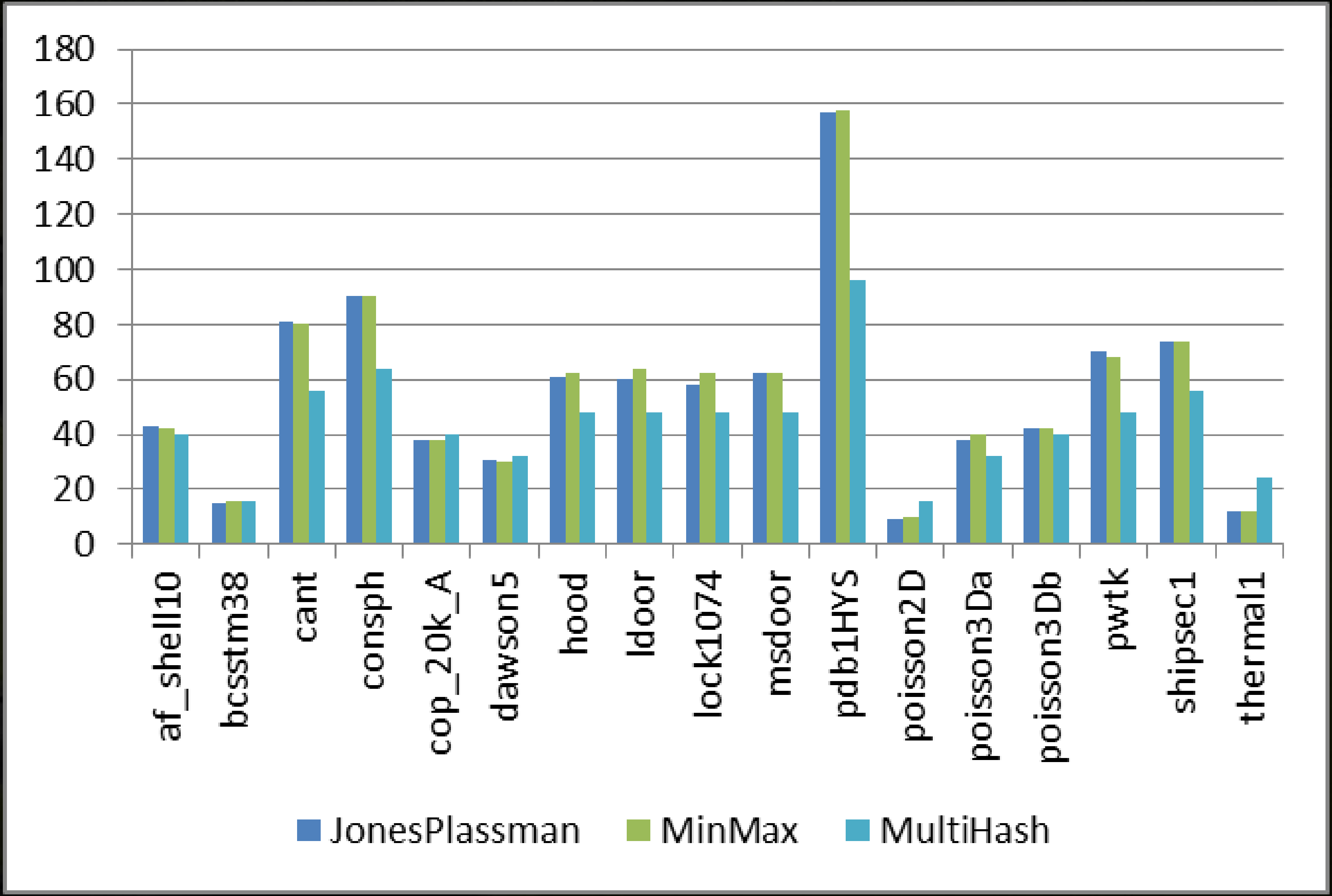
- Combine all 2-colorings – completes in 1 round!
- Creates well-balanced graph colorings
- Empirically: produces better colorings than Luby-Jones – not sure why



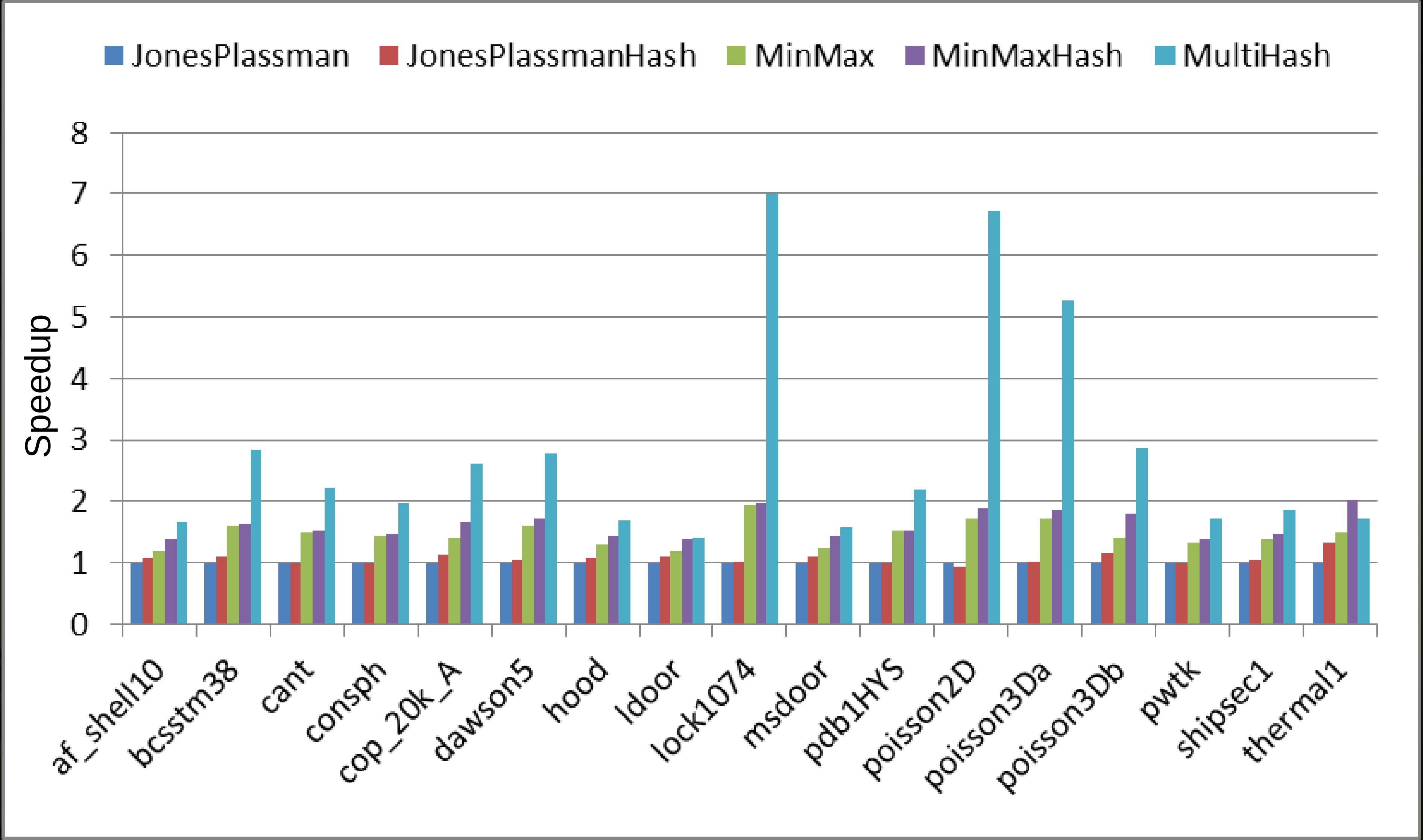
100% Coloring Results



Number of Colors (100% Coloring)

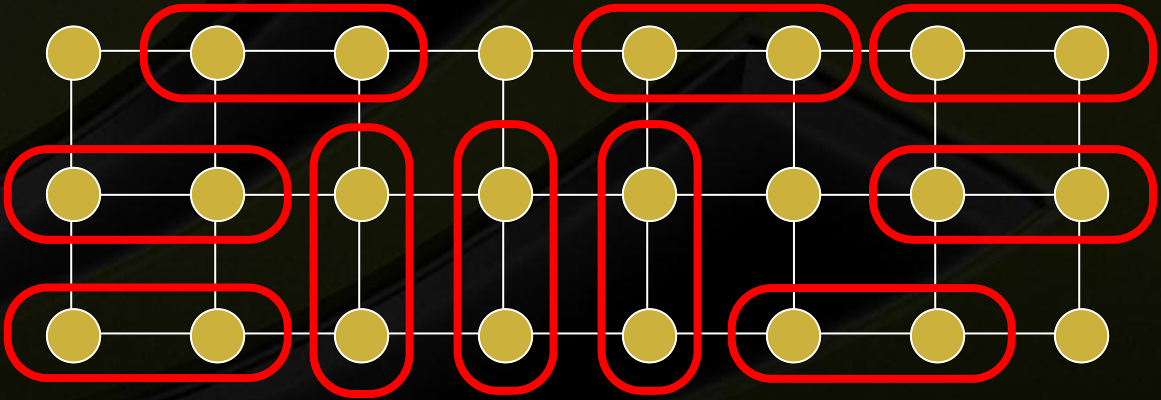


95% Coloring Results

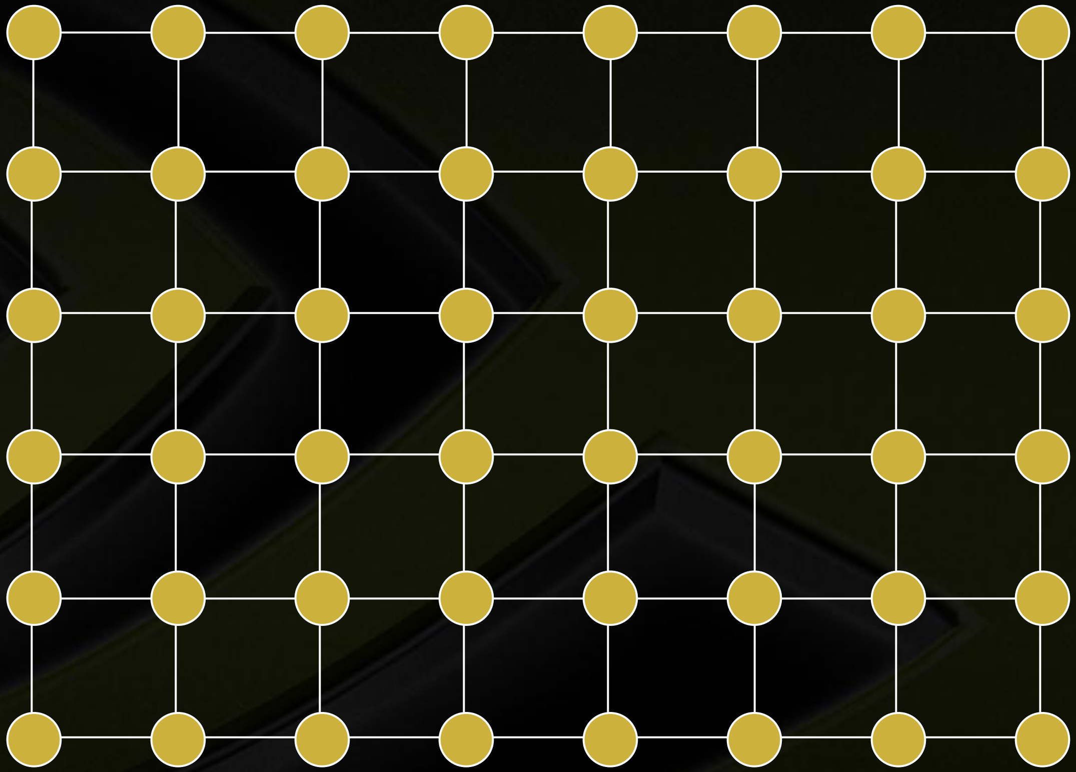


Graph Matching

- Set of edges such that no two edges share a vertex
- “Maximum matching” – matching the includes the largest number of edges
- Equivalent: Independent set on dual of graph
  - independent *pairs* of connected vertices

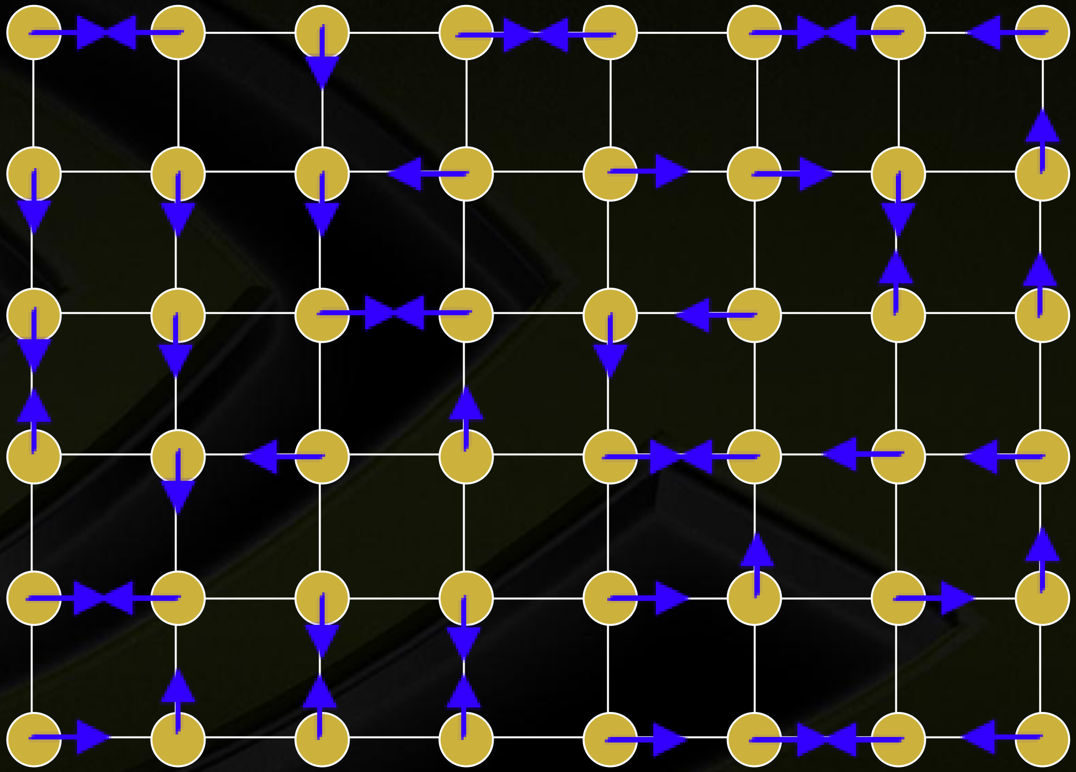


One-Phase Handshaking



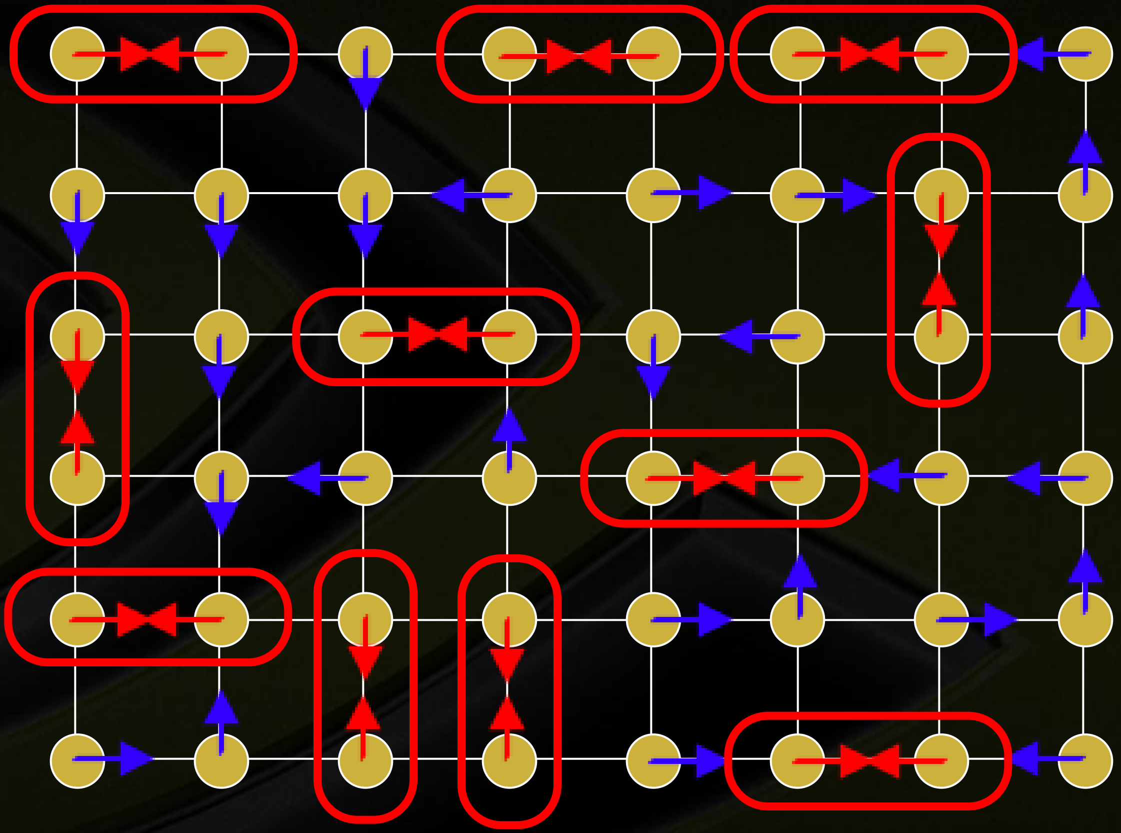
One-Phase Handshaking

- **Each vertex extends a hand to its strongest neighbour**



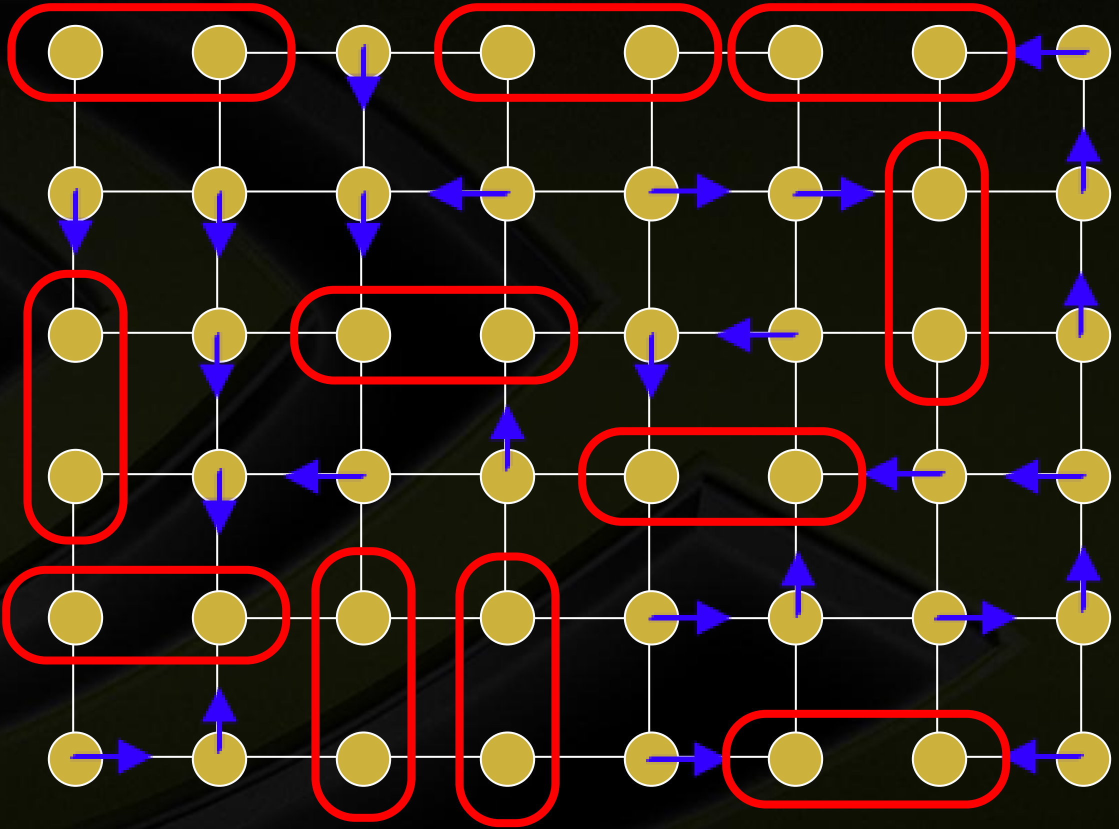
Set aggregates

- **Each vertex checks if its strongest neighbour extended a hand back**

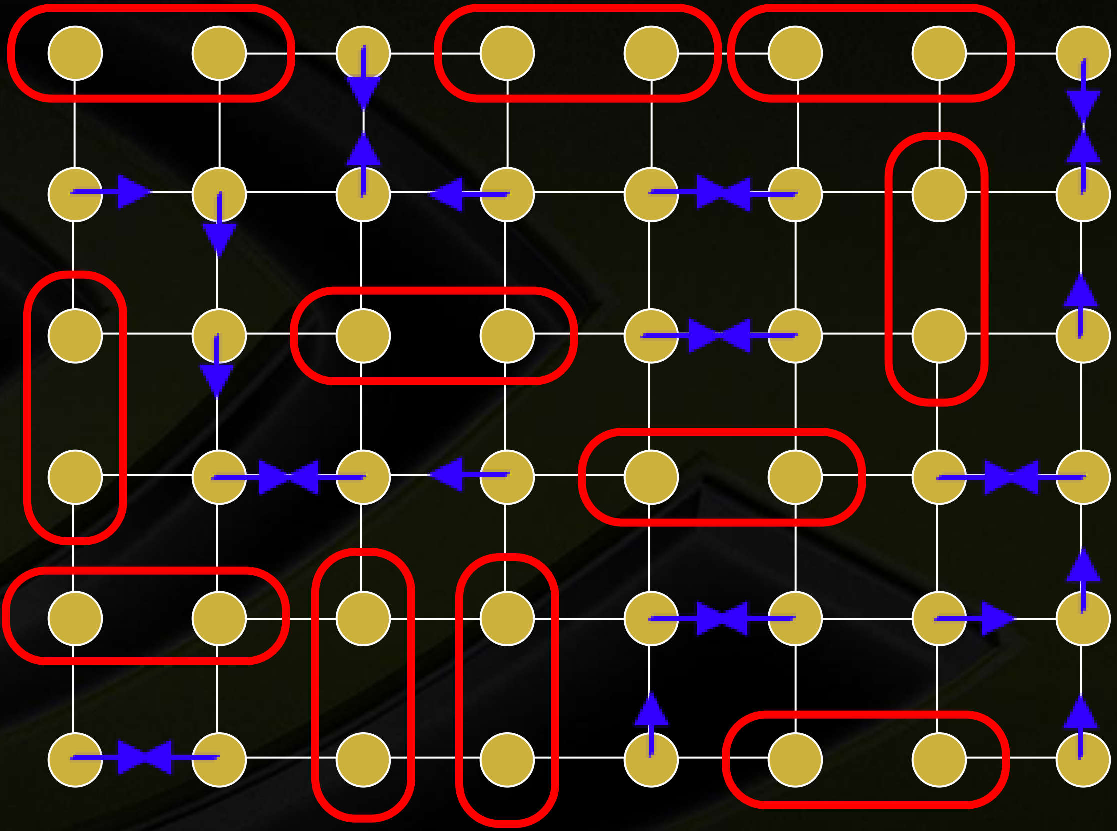


One-Phase Handshaking

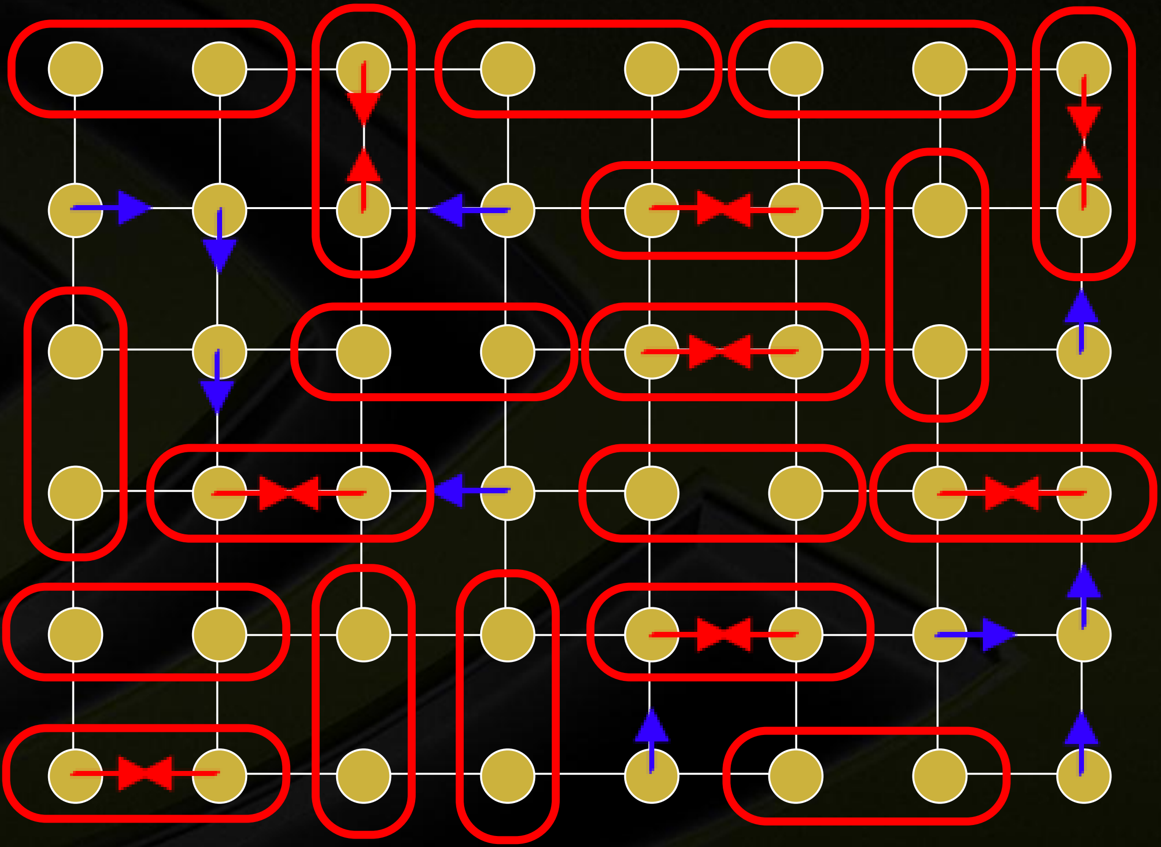
- **Repeat with unmatched vertices**



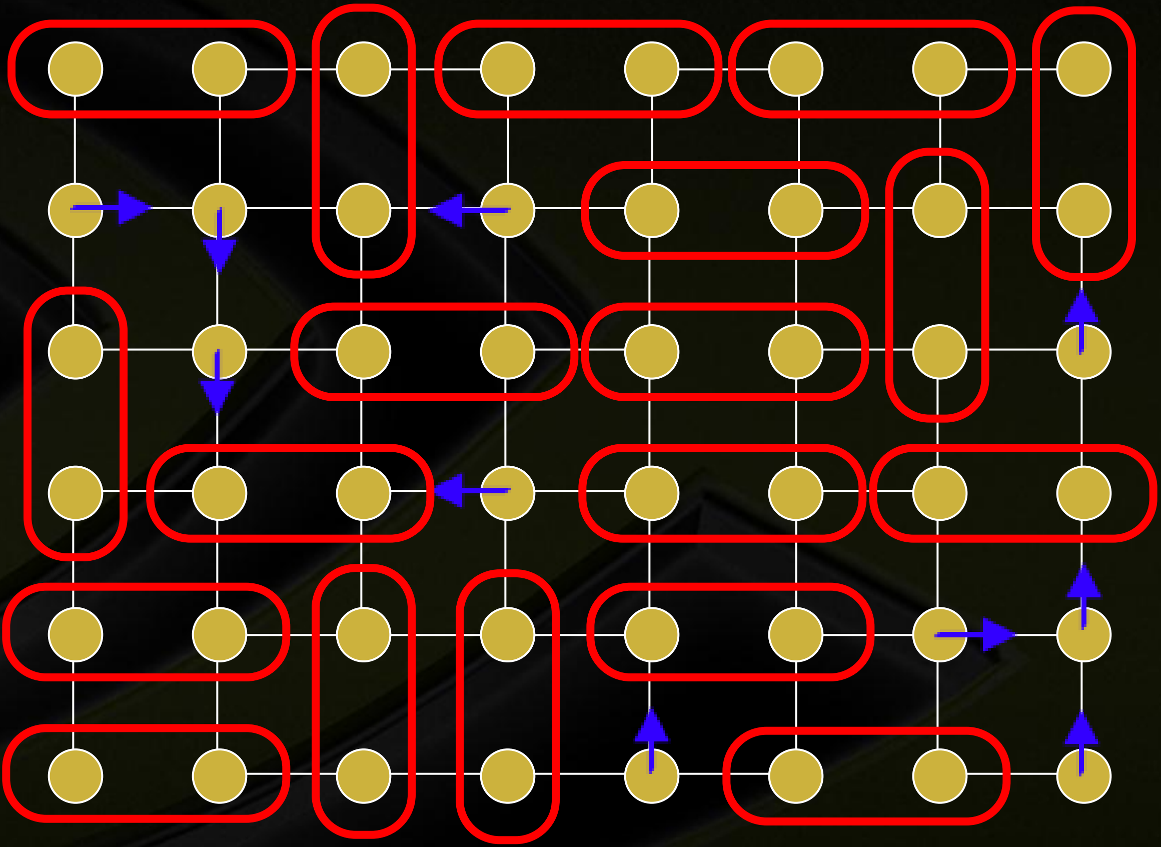
One-Phase Handshaking



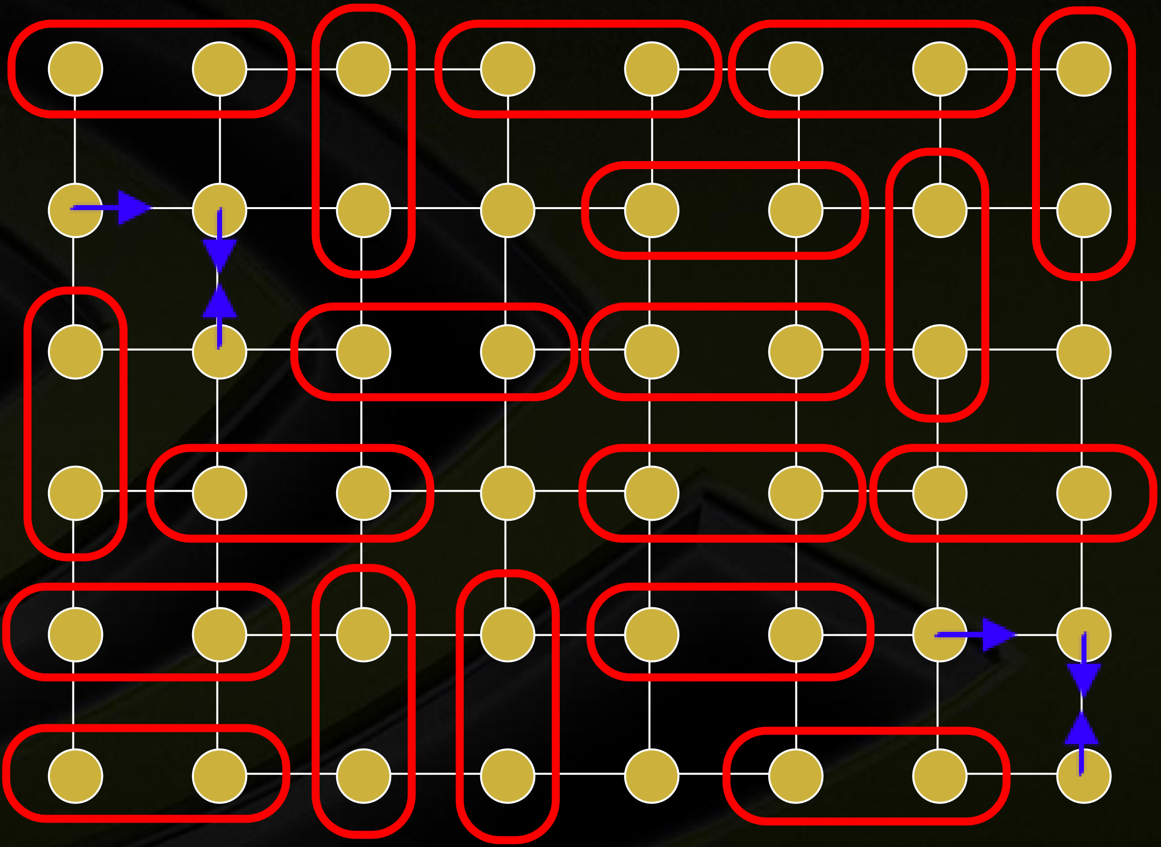
One-Phase Handshaking



One-Phase Handshaking



One-Phase Handshaking



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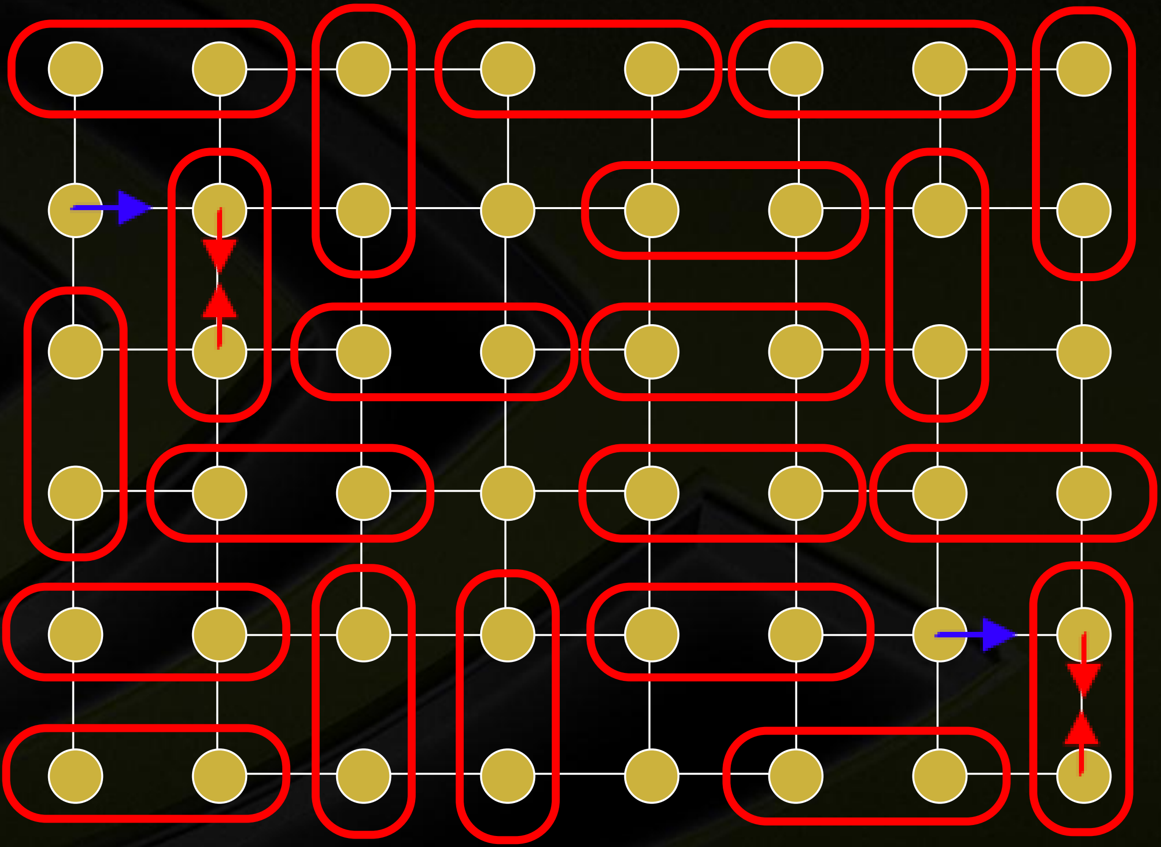
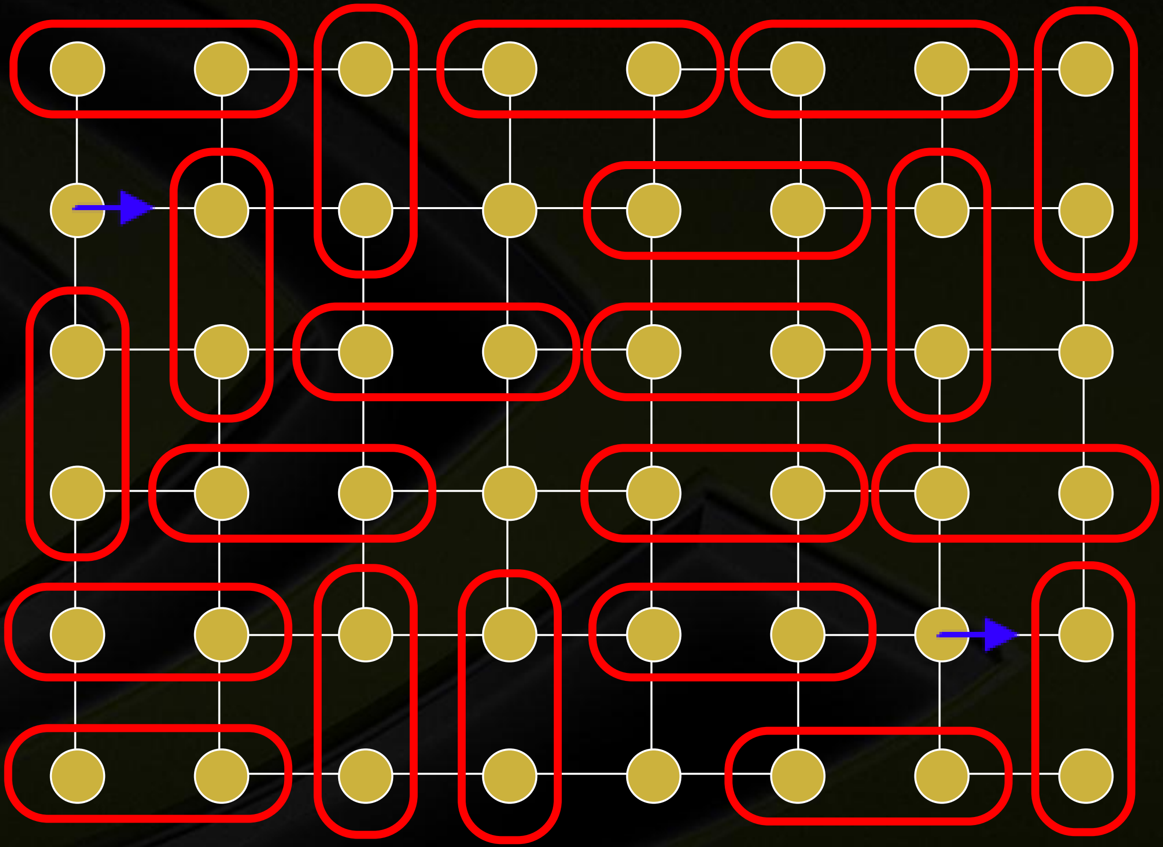
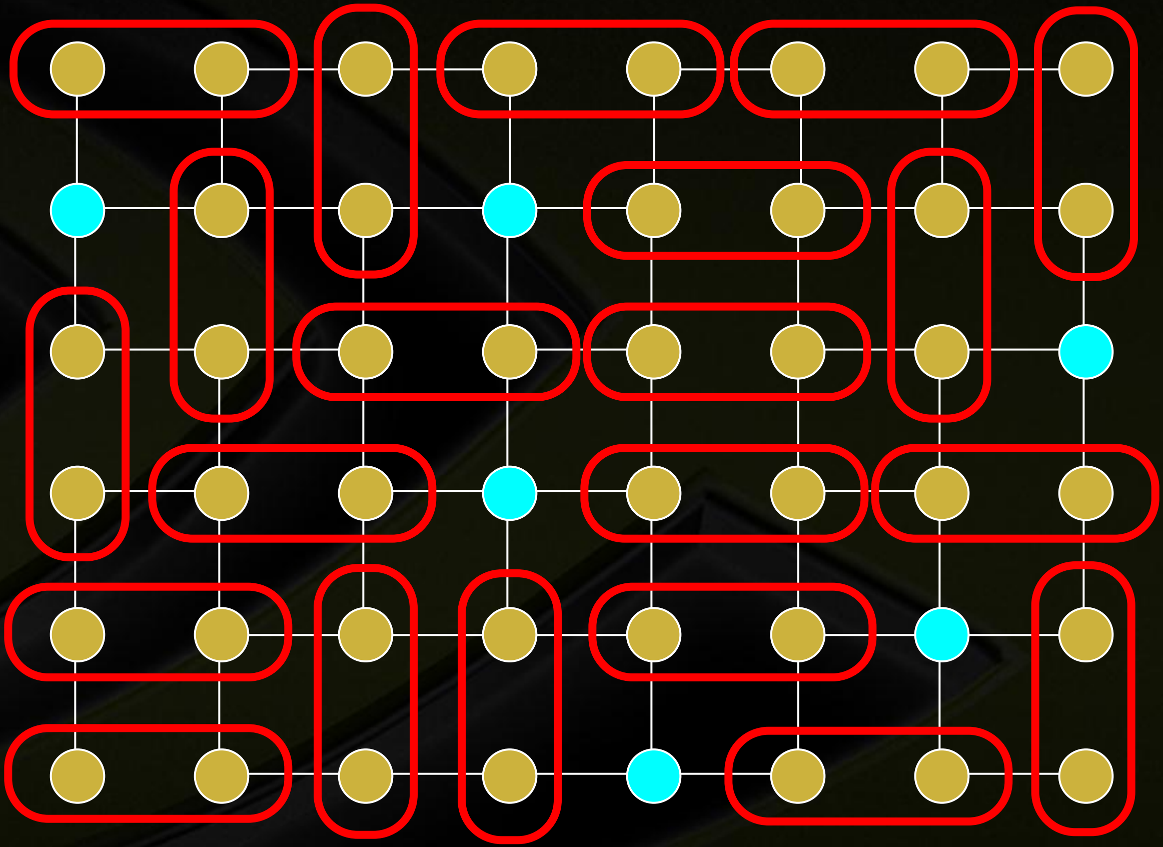


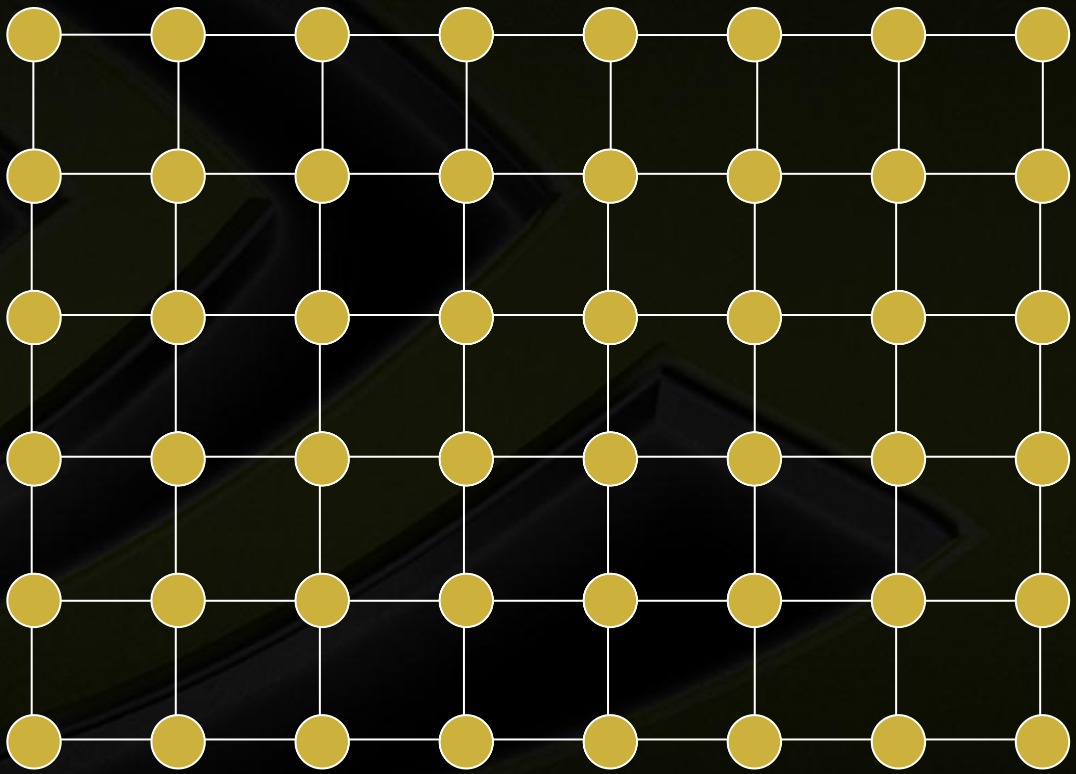
One-Phase Handshaking



One-Phase Handshaking

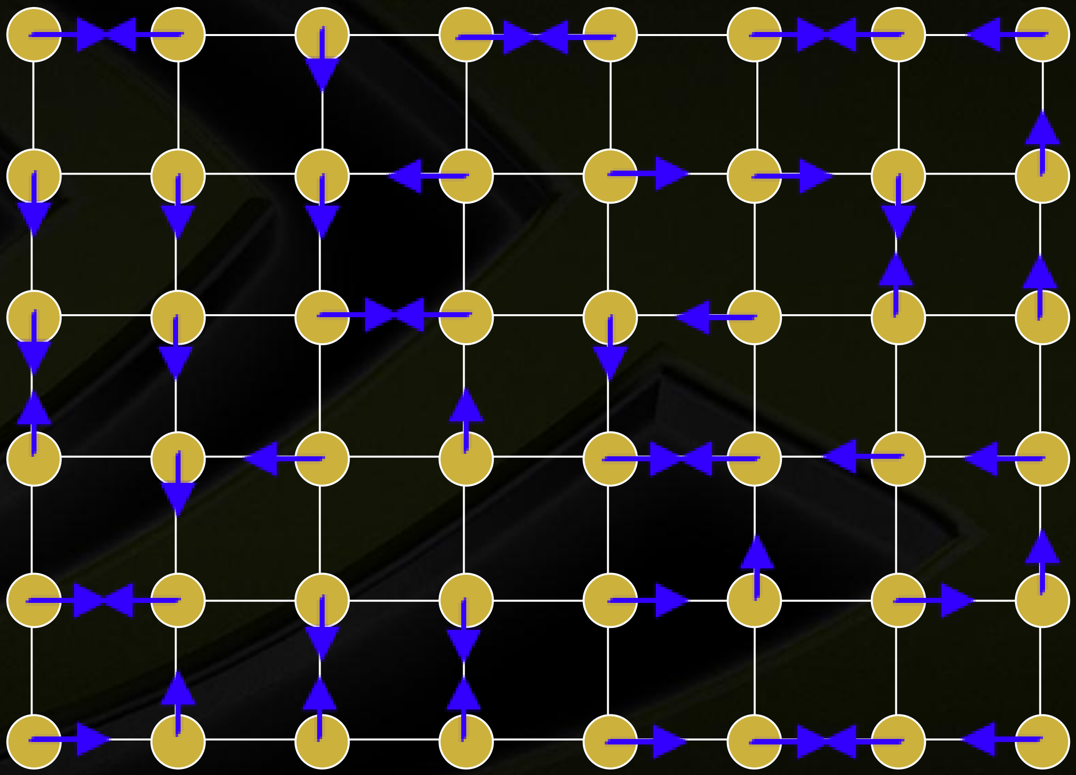


Two-Phase Handshaking



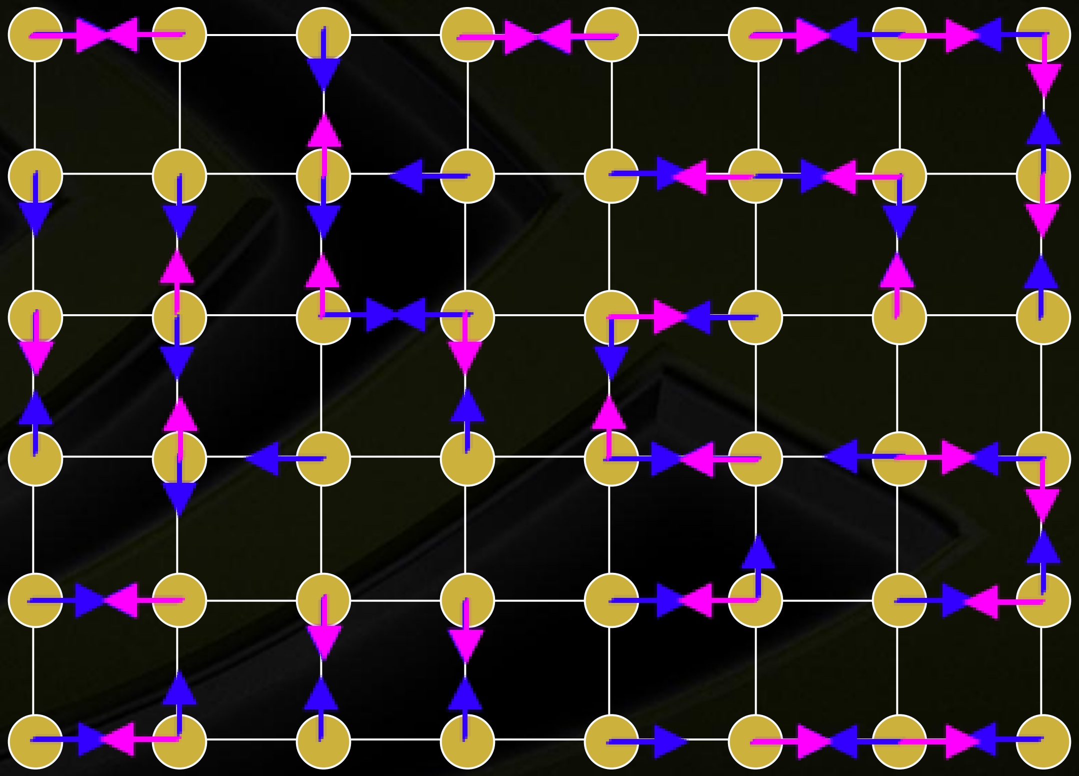
Two-Phase Handshaking

- **Extend a first hand to your strongest neighbour**



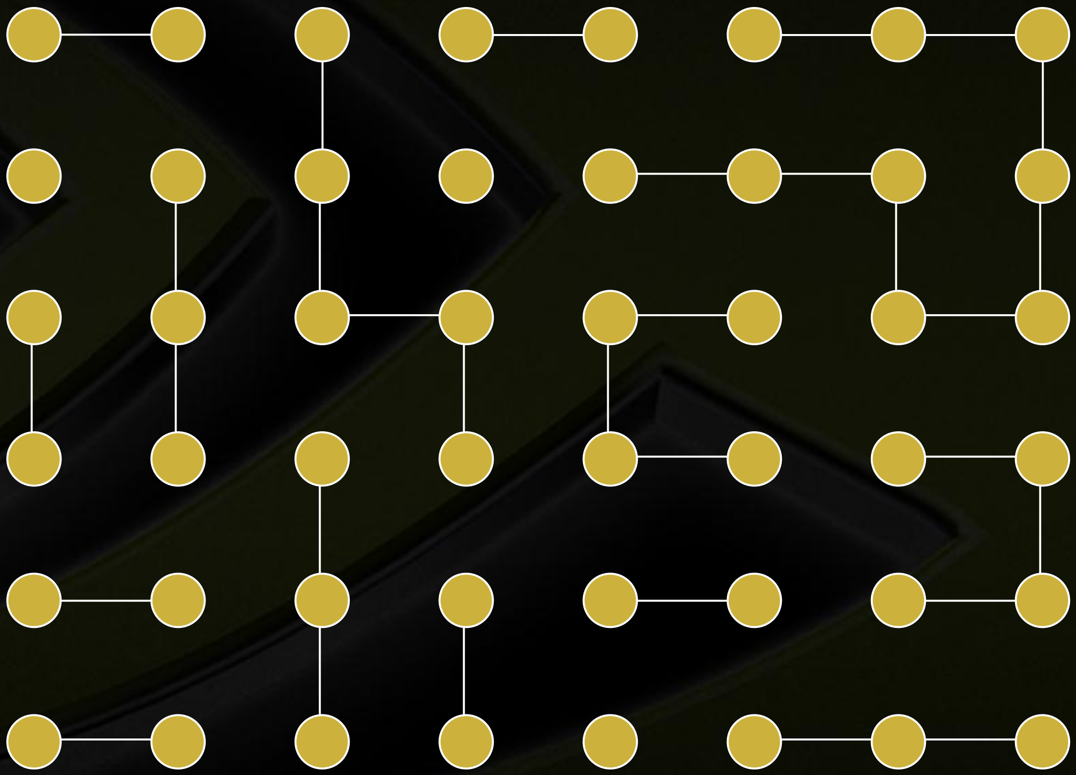
Two-Phase Handshaking

- **Extend a second hand to the strongest vertex among those who gave a hand to you**



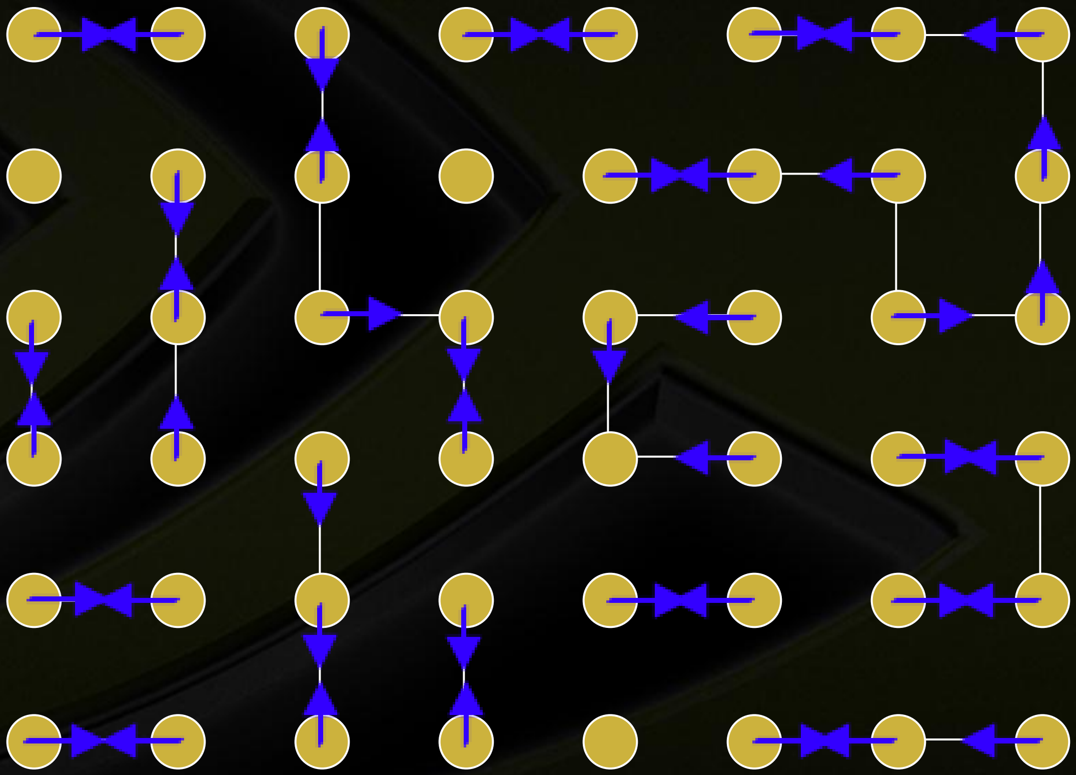
Two-Phase Handshaking

- **Keep only edges which have a handshake**
- **New graph has maximum degree 2**



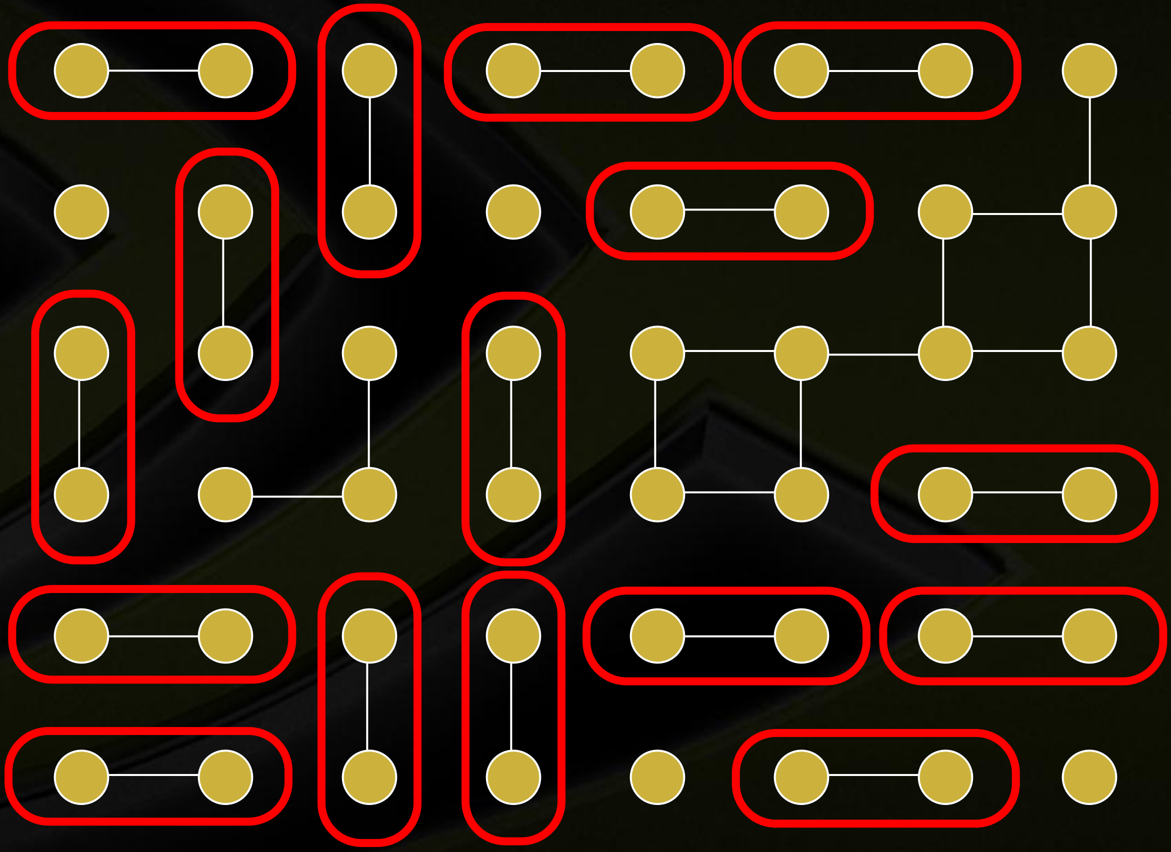
Two-Phase Handshaking

- **Now do one-phase handshaking**



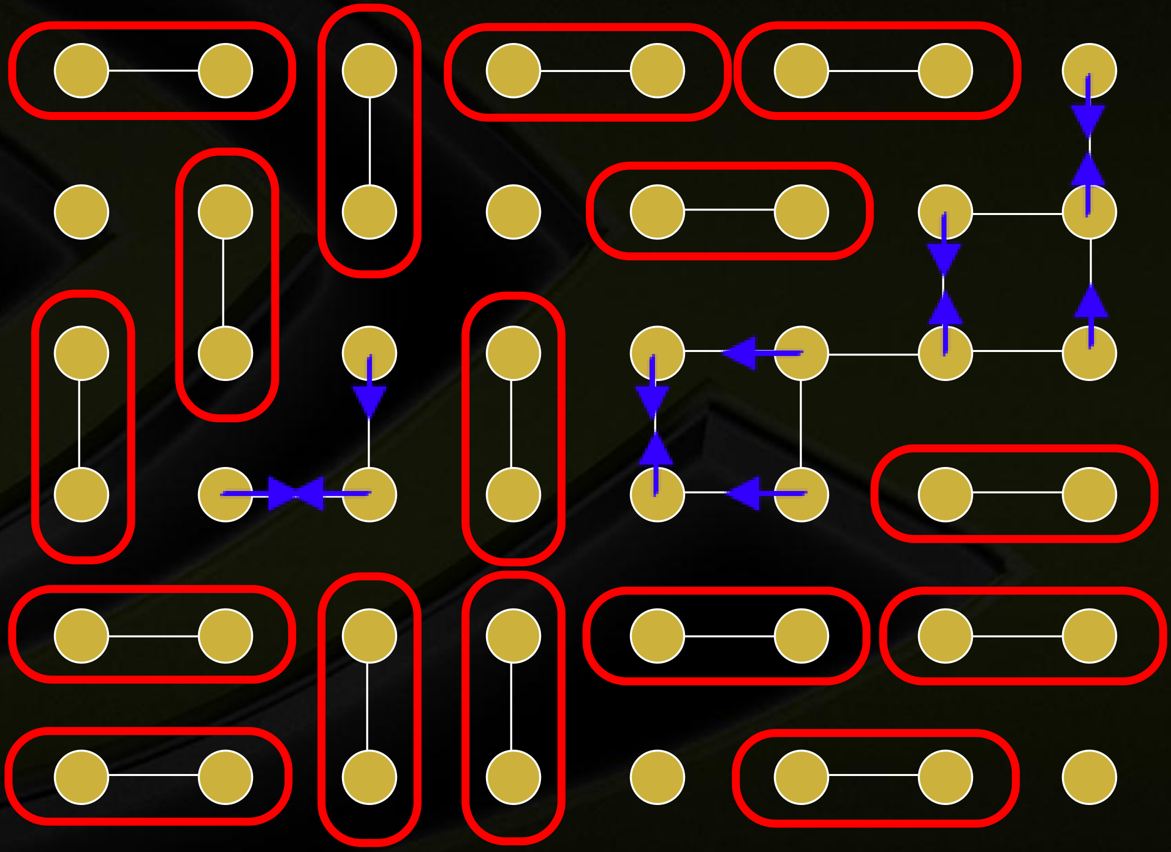
Two-Phase Handshaking

- **Find perfect matches**



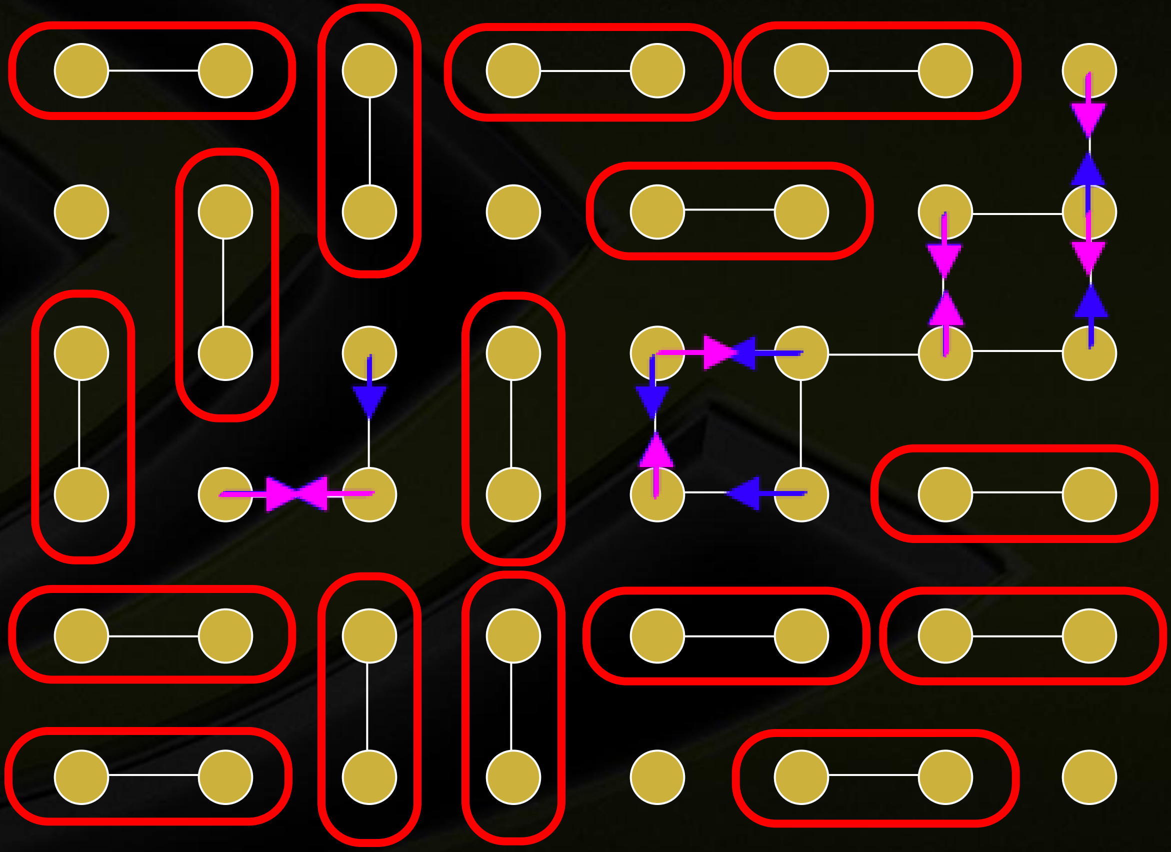
Two-Phase Handshaking

- **Repeat**



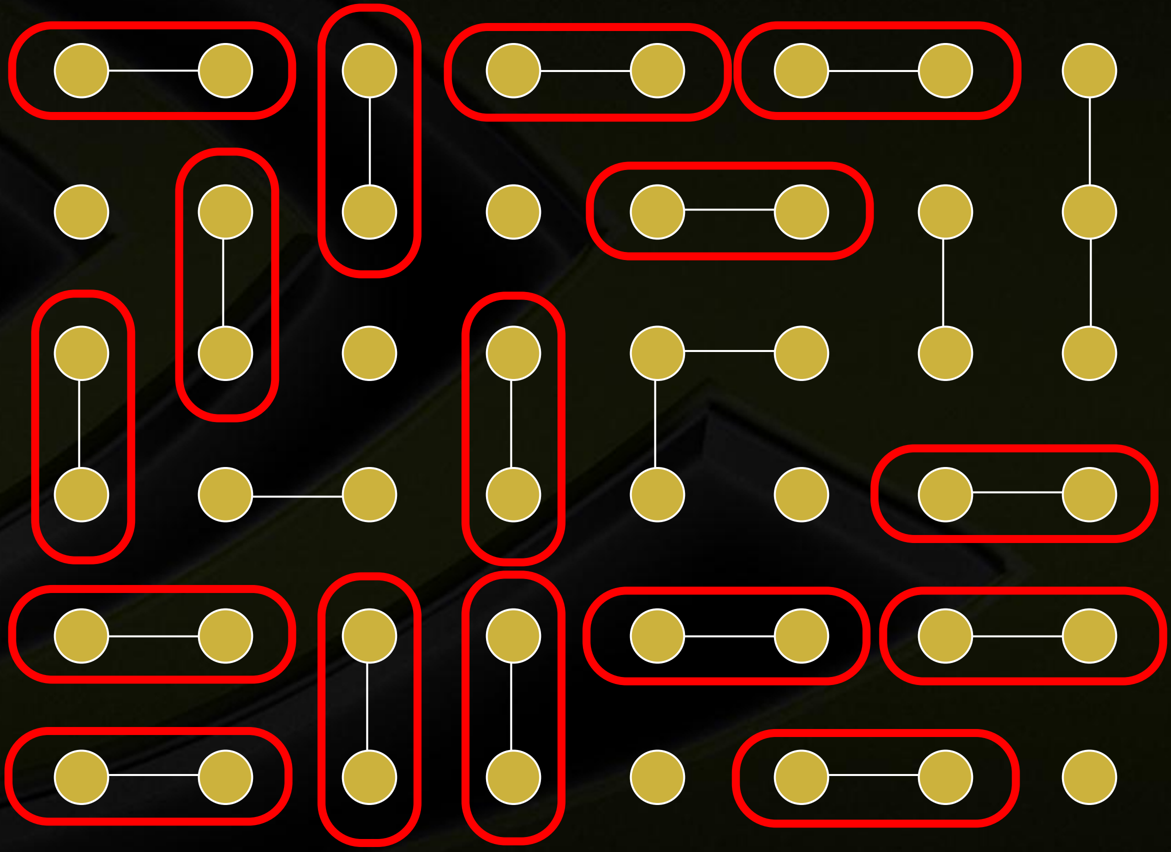
Two-Phase Handshaking

- **Repeat**



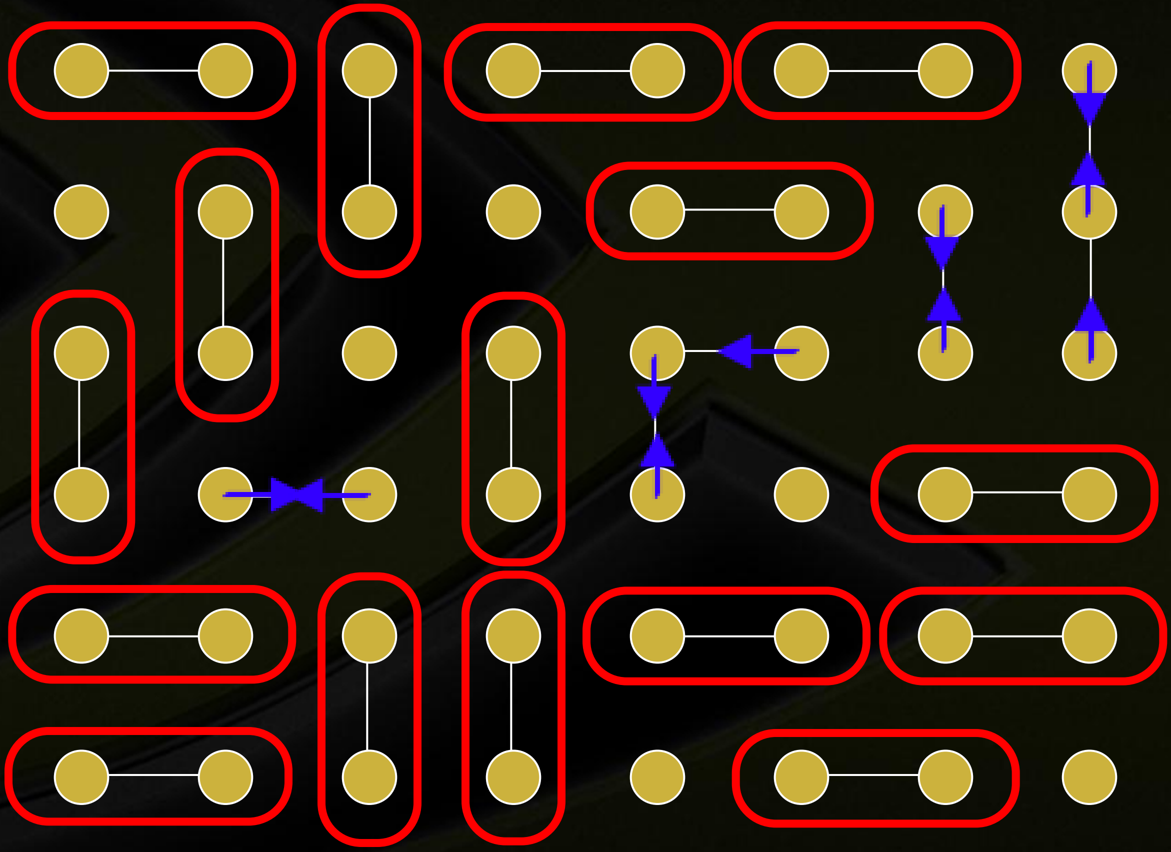
Two-Phase Handshaking

- **Repeat**



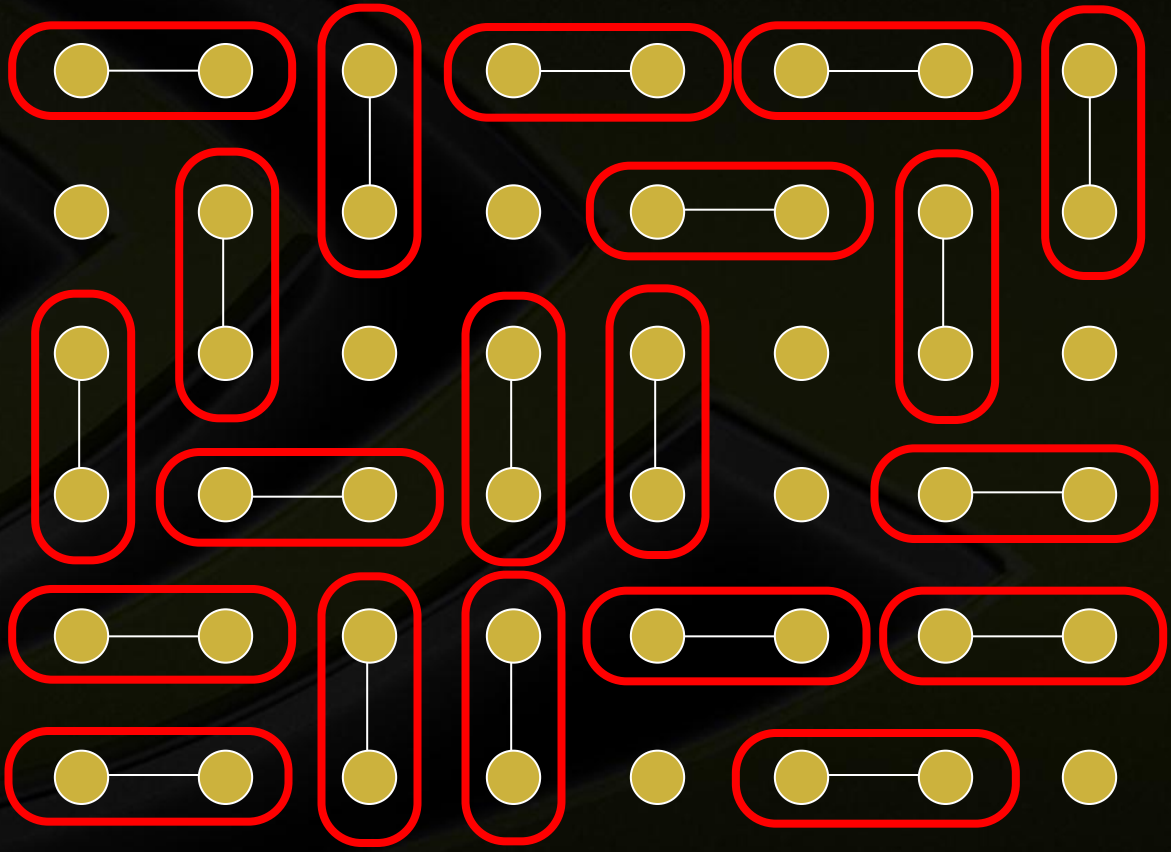
Two-Phase Handshaking

- **Repeat**



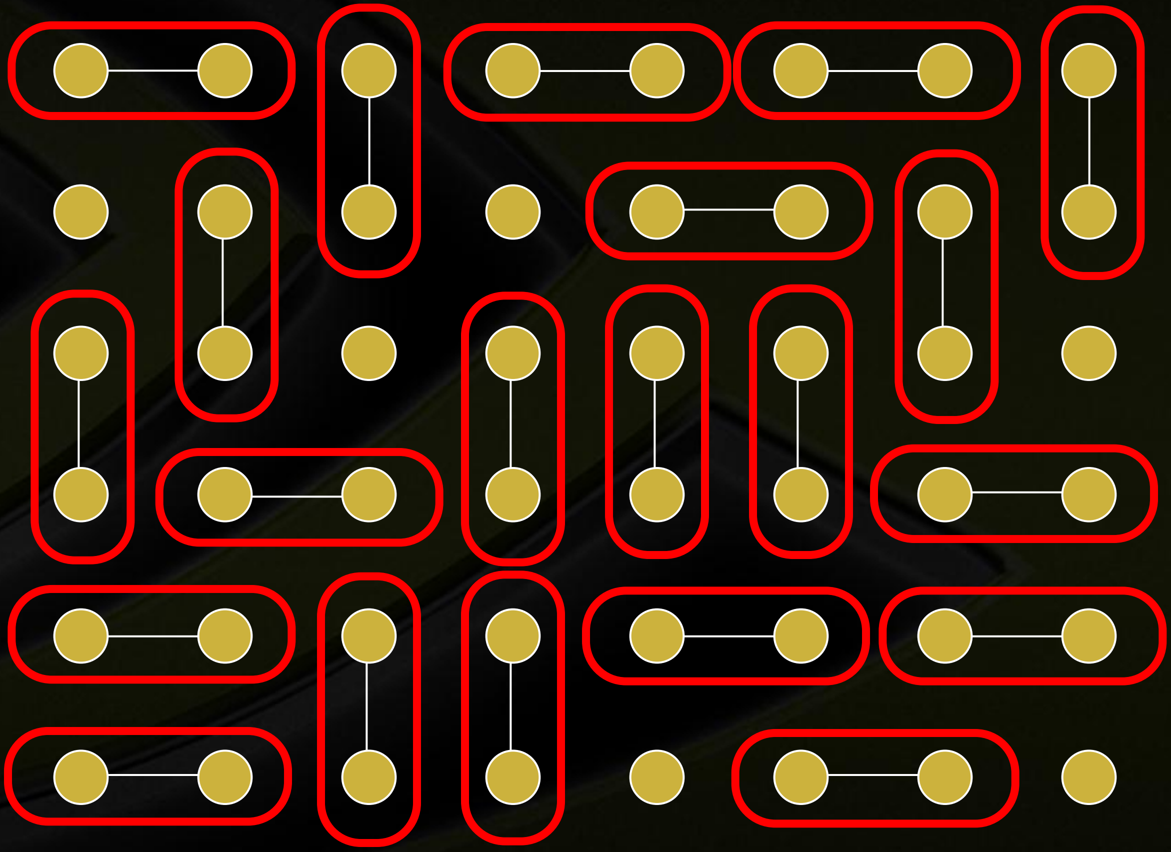
Two-Phase Handshaking

- **Repeat**

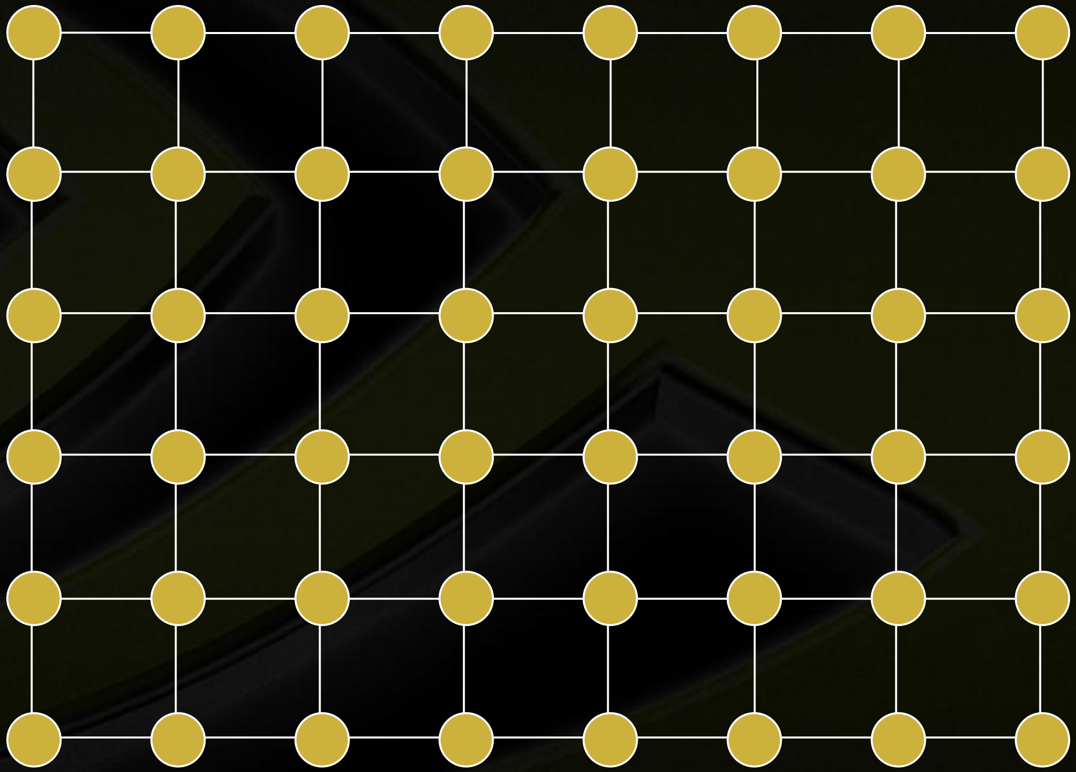


Two-Phase Handshaking

- **Repeat**

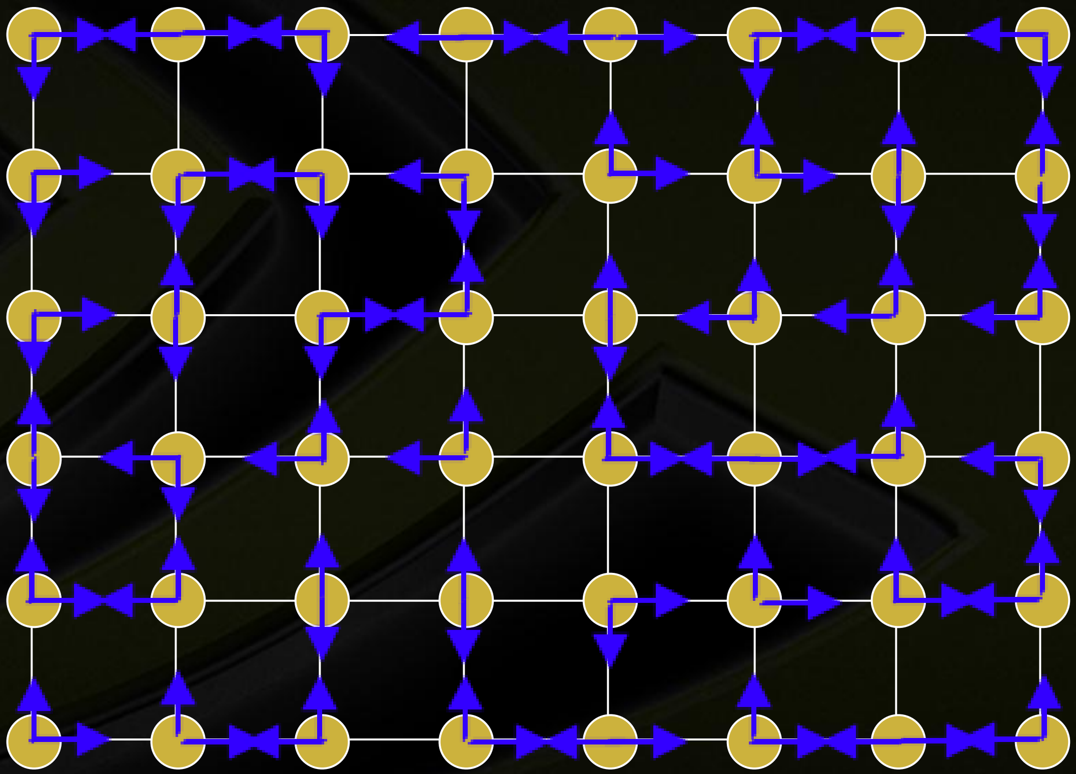


N-Way Handshaking



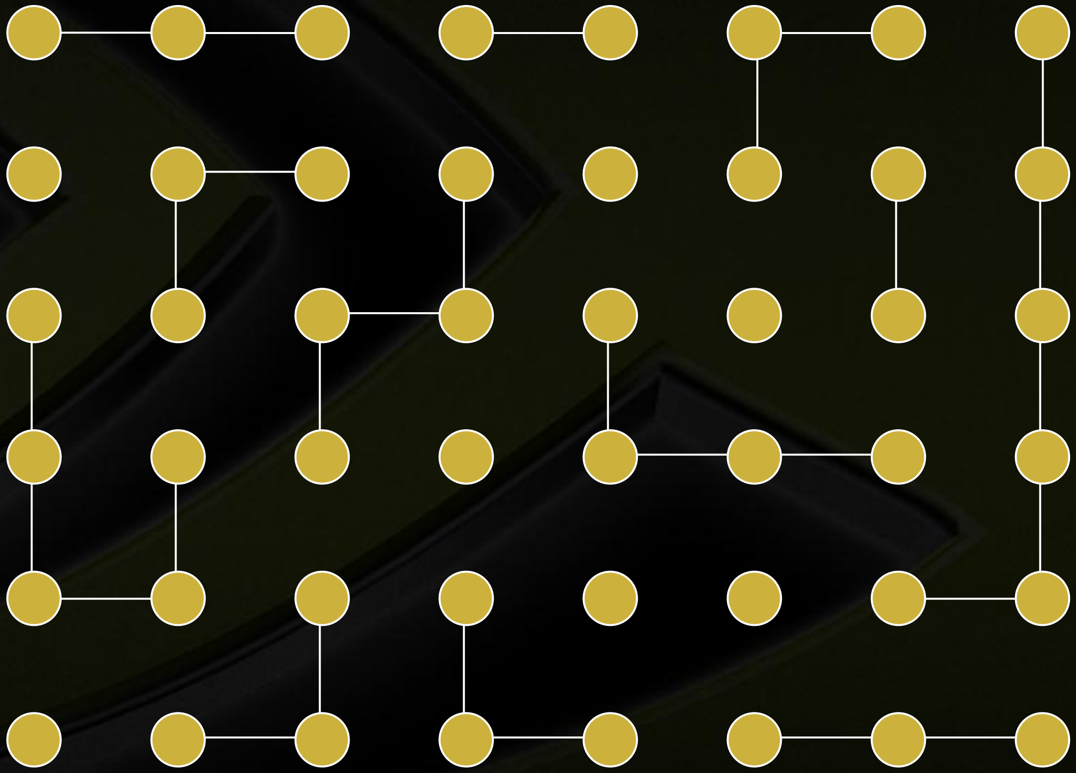
N-Way Handshaking

- Extend N hands at once (N=2)
- Similar to first 2 steps of two-phase, but in a single step



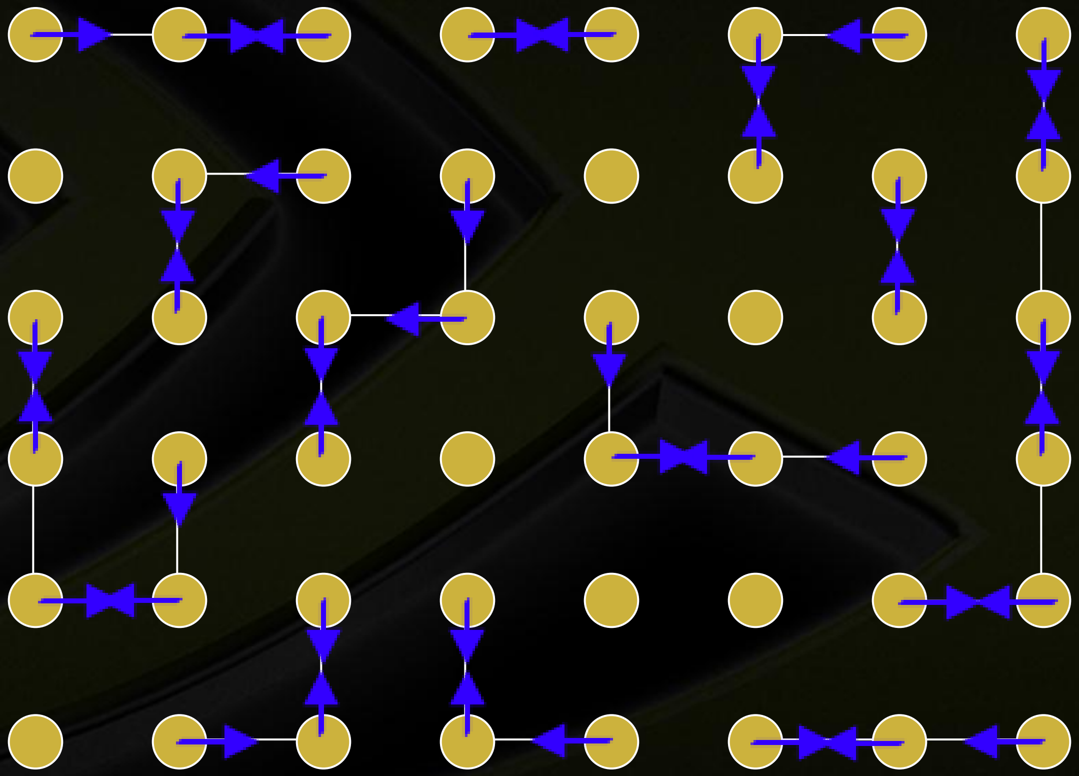
N-Way Handshaking

- Discard edges without a match
- Resulting graph has max degree N (N=2)



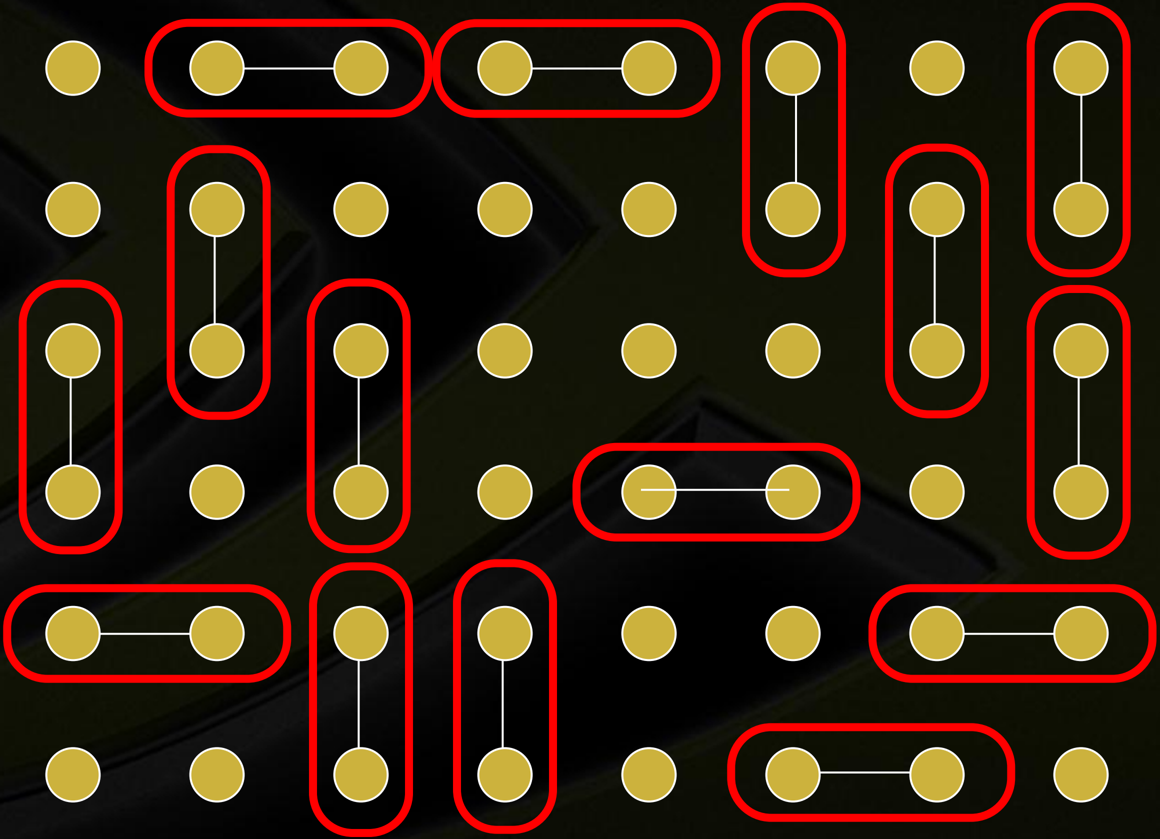
N-Way Handshaking

- **Now do one-phase**



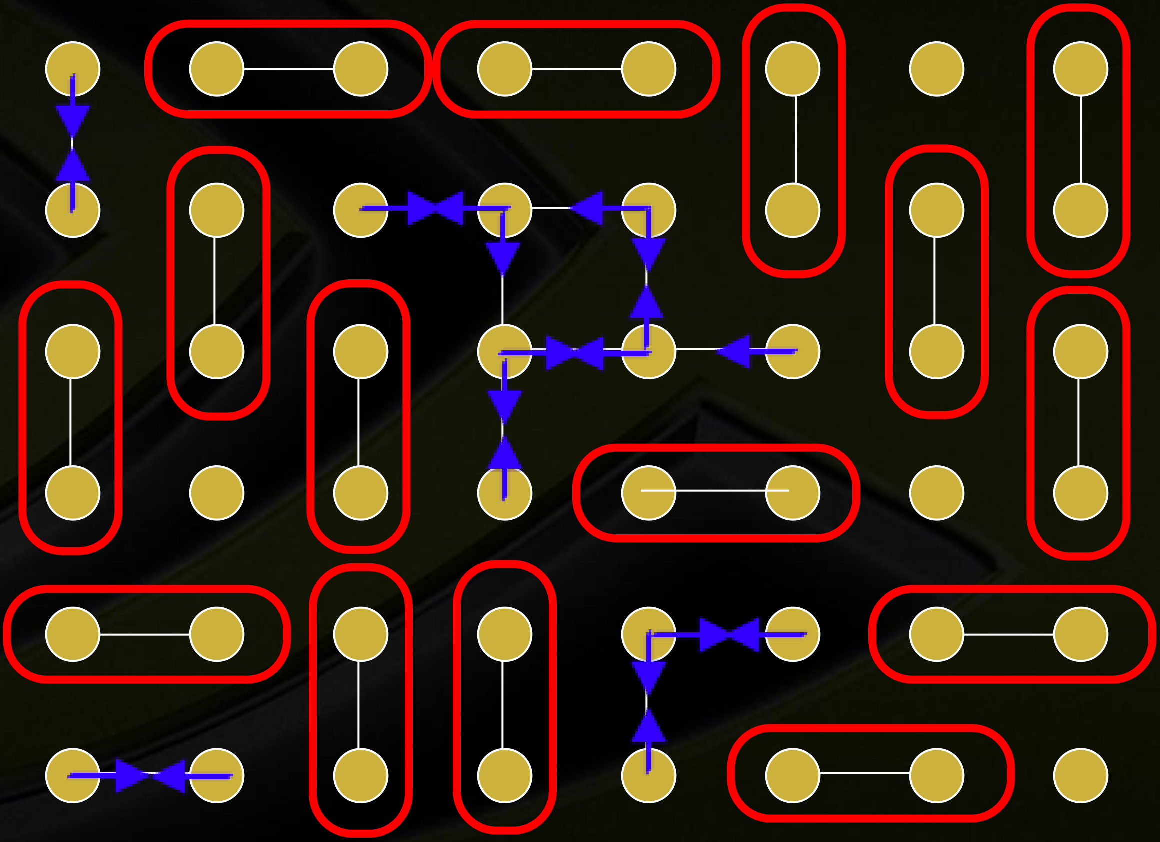
N-Way Handshaking

- **Select perfect matches**



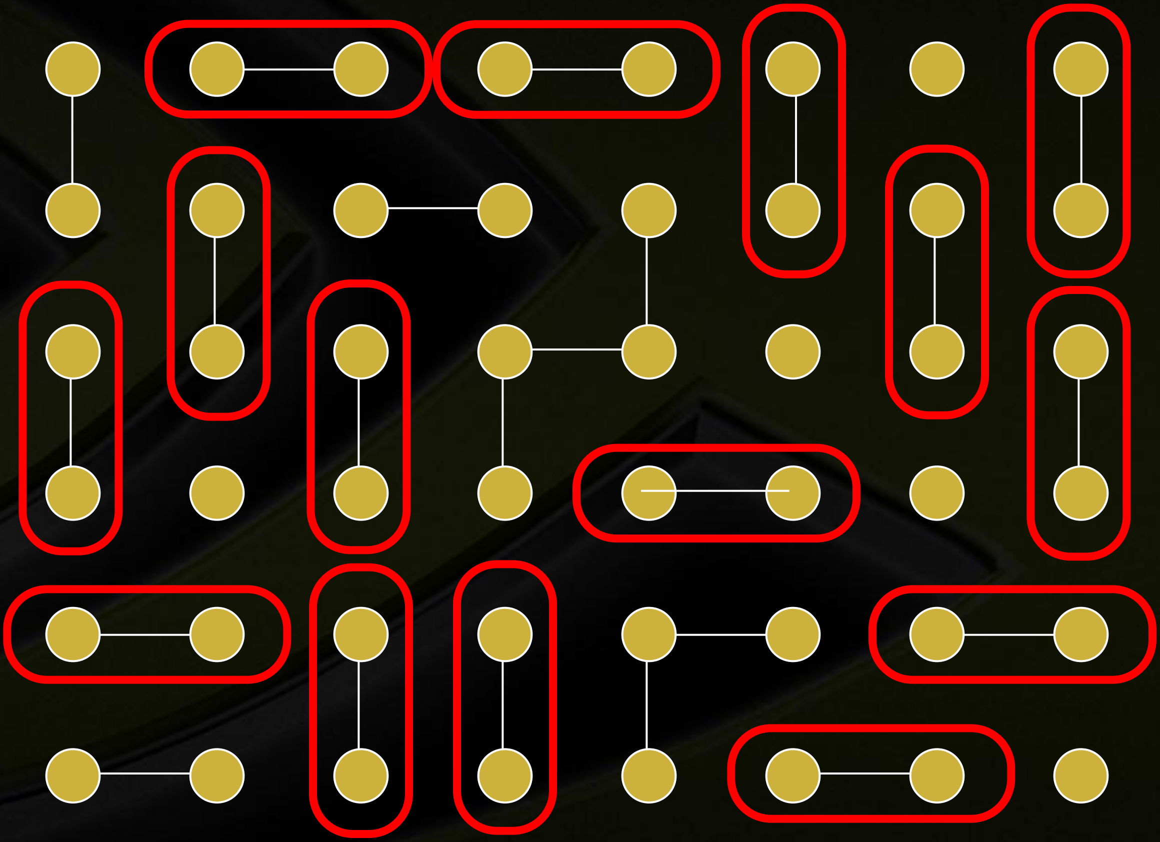
N-Way Handshaking

- **Repeat**



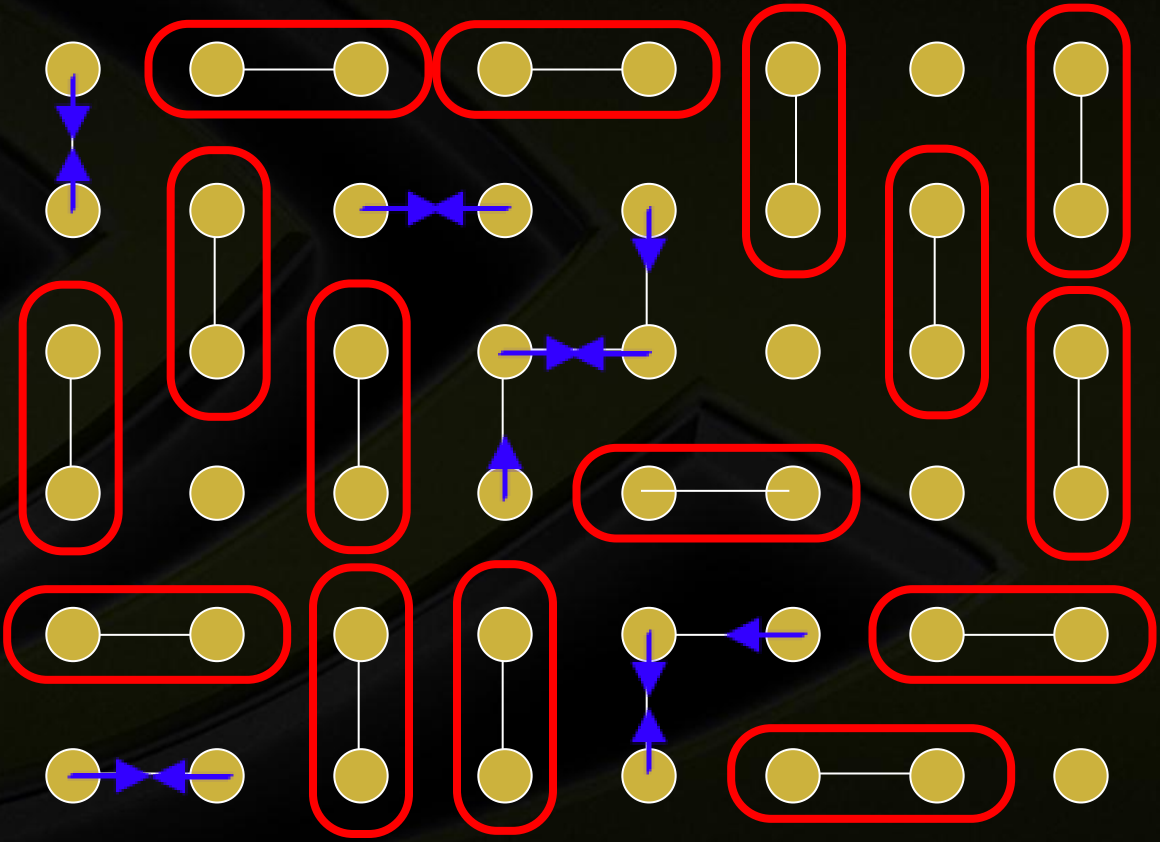
N-Way Handshaking

- **Repeat**



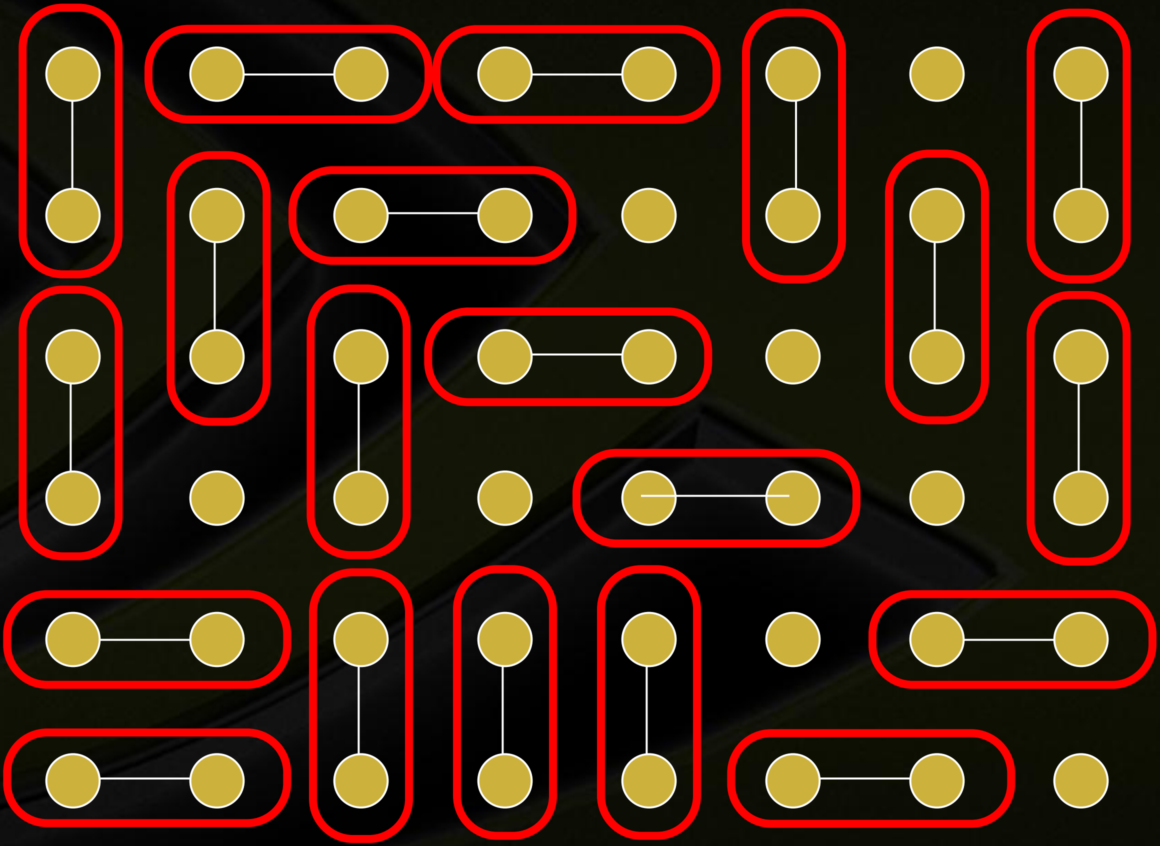
N-Way Handshaking

- **Repeat**

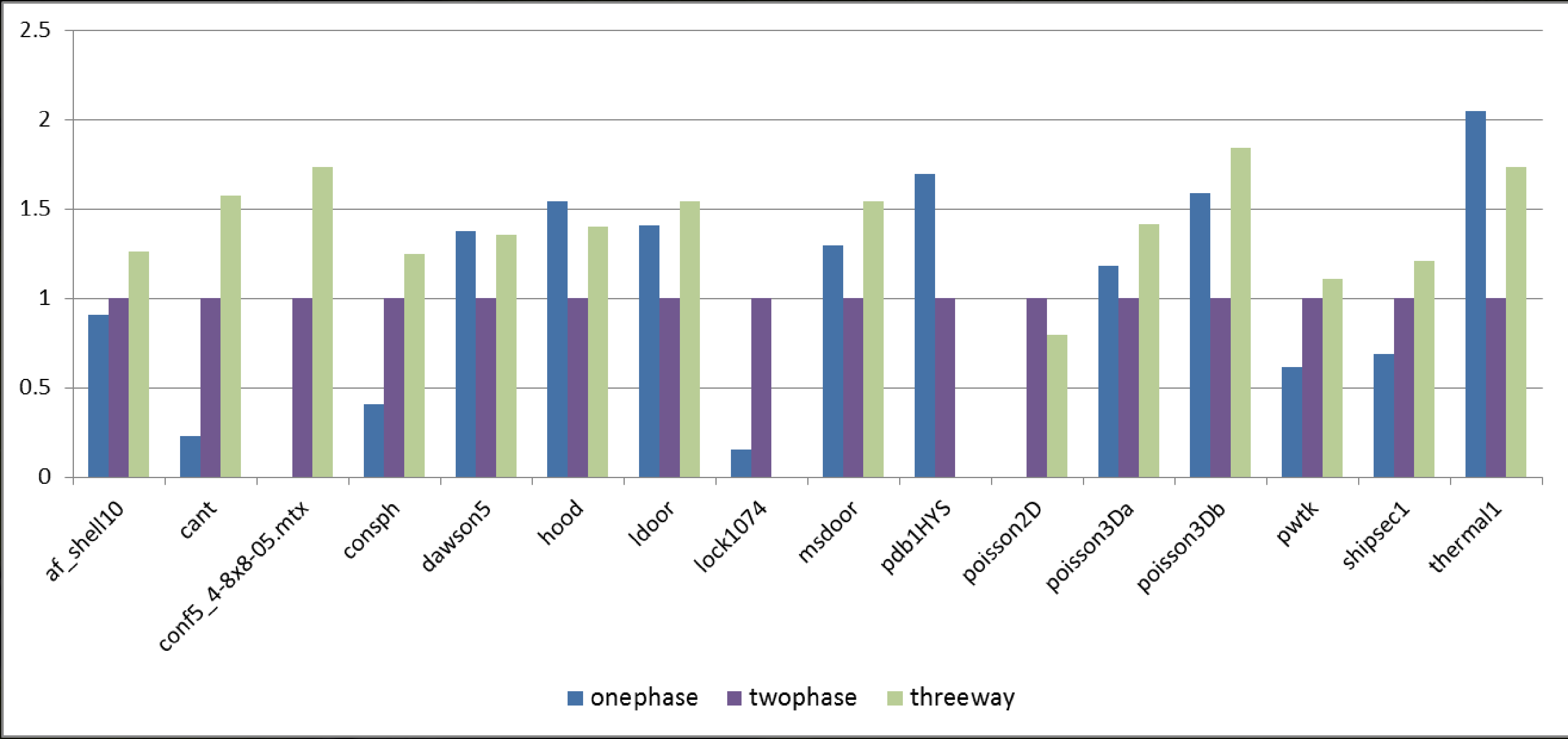


N-Way Handshaking

- Repeat



Graph Matching Performance (90% Matching Target)



Conclusion

- Graph Coloring and Matching algorithms can be highly data parallel
- Key optimizations:
  - More work per thread, fewer global synchronizations
  - Replace random numbers with hash functions

- One view: recast in terms of generalized Sparse Matrix-Vector product (SpMV)

```
For each row (in parallel)
    Visit each neighbor, compute something
    Compute reduction
    Write out single result
```

Questions?

- Tech report and source code with lots more details is forthcoming
- Thanks to entire NVAMG team